Interpolation, Correlation and NIST Standards

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Light is OSRAM

Spectral irradiance measurement

$$Spectrum = Assignment \bullet \frac{Test}{Calibration} \bullet \frac{\langle errors \rangle}{\sqrt{n}} \qquad n \to \infty$$

Assume all calibration have been applied so that all errors are zero mean.

$$Spectrum = Assignment \bullet \frac{Test}{Calibration}$$

I will consider only the uncertainty in the result that is due to the assignment.

$$Spectrum = Assignment \bullet M$$

Perfect measurement and calibration
$\underline{u}_c(M)=0$

$$\frac{\Delta Spectrum}{Spectrum} = \frac{\Delta Assignment}{Assignment}$$

Relative change / Relative uncertainty

$$\Delta\% Spectrum = \Delta\% Assignment$$

Uncertainty propagated as percent.

Naive analysis of uncertainty

$$Y = f(X_1, X_2, \ldots, X_n)$$

Measurement Equation

$$u_c^2(y) = \sum_{i=1}^{N} \left(\frac{\partial f}{\partial x_i} \right)^2 u^2(x_i)$$

(10)

$$u_c^2(y) = \sum_{i=1}^{N} \left[c_i u(x_i) \right]^2$$

(11a)

$$u_i(y) \equiv \left| c_i \right| u(x_i)$$

(11b)

$$c_i = \frac{u_i(y)}{u(x_i)}$$

By the book

$$Lumen = f_{lm}(S_1, S_2, \ldots S_n)$$

$$CIEx = f_{CIEx}(S_1, S_2, \ldots S_n)$$

$$CIEy = f_{CIEy}(S_1, S_2, \ldots S_n)$$

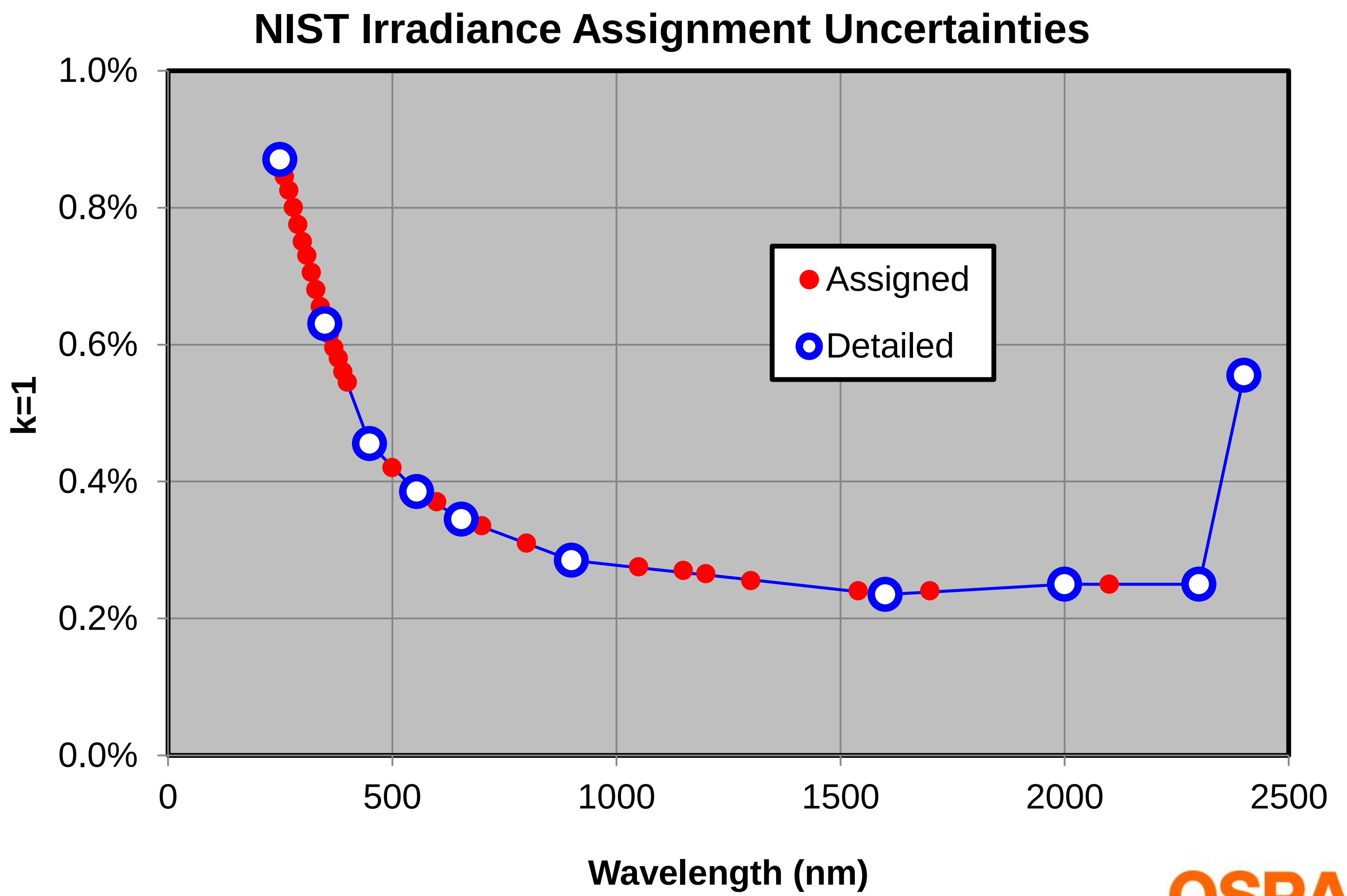
S_i = measured value at wavelength i
i = 360 nm to 830 nm

$$u_i(Lumen) = c_i \bullet u(S_i)$$

$$u_i(Lumen) = c_i \bullet u(A_i)$$

A_i = Assigned value at wavelength i
$u(A_i)$ is interpolated

Linear Interpolation of Uncertainties



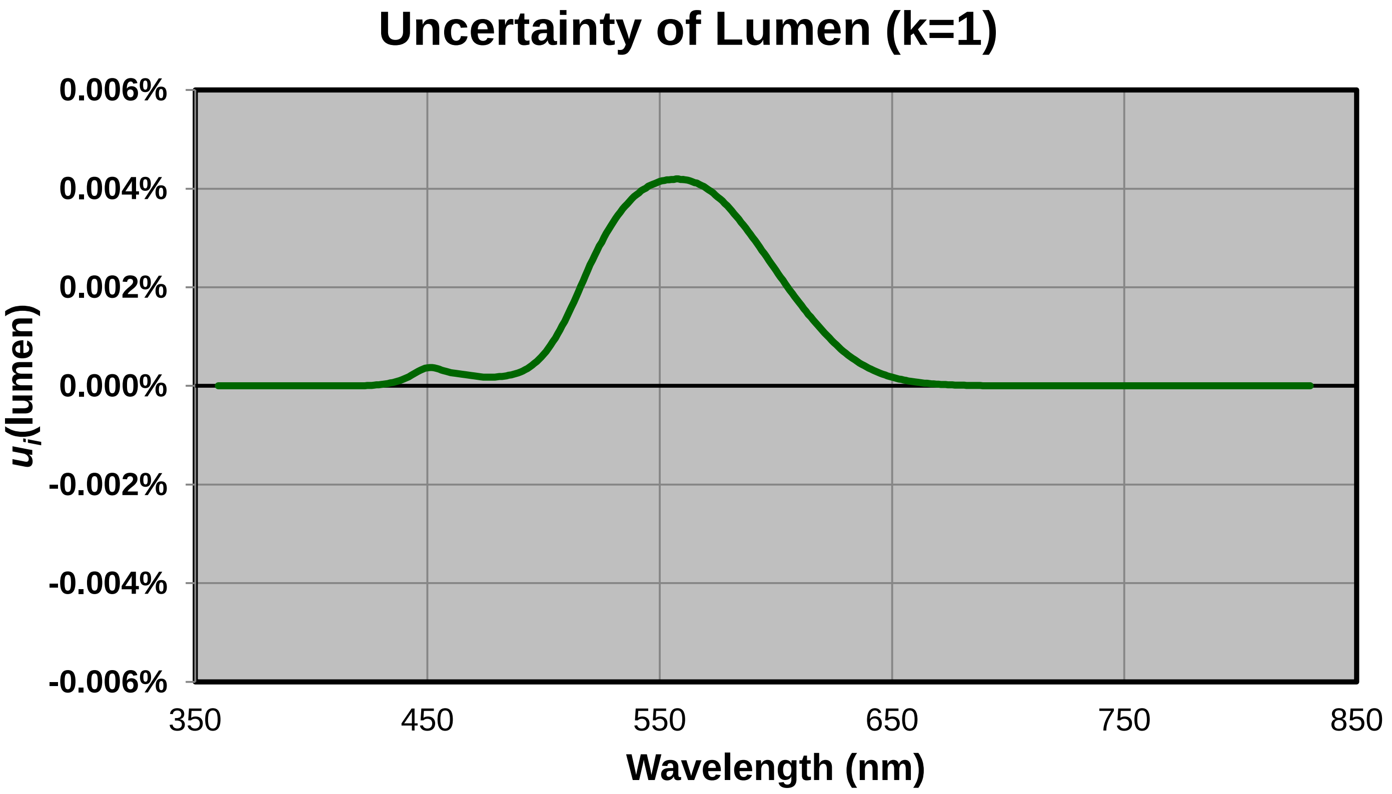
Lumen Measurement Equation(s)

$$lm = 683 \bullet \begin{pmatrix} S_{360} \\ S_{361} \\ \ldots \\ S_{830} \end{pmatrix}^T \begin{pmatrix} \overline{Y}_{360} \\ \overline{Y}_{361} \\ \ldots \\ \overline{Y}_{830} \end{pmatrix}$$

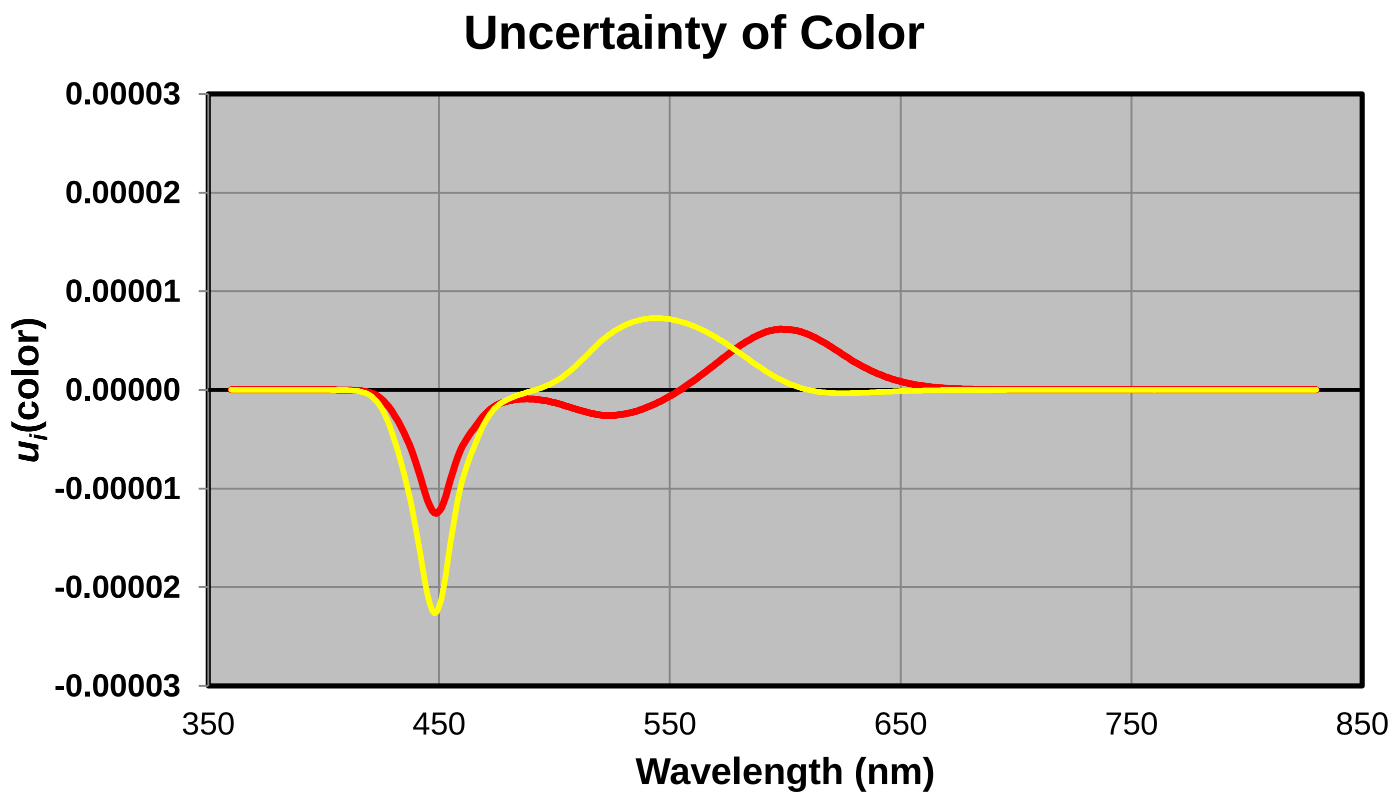
$$\begin{pmatrix} S_{360} \\ S_{361} \\ \ldots \\ S_{830} \end{pmatrix} = \begin{pmatrix} A_{360} \\ A_{361} \\ \ldots \\ A_{830} \end{pmatrix} \bullet \begin{pmatrix} T_{360} \\ T_{361} \\ \ldots \\ T_{830} \end{pmatrix} \bullet \begin{pmatrix} 1/Cal_{360} \\ 1/Cal_{361} \\ \ldots \\ 1/Cal_{830} \end{pmatrix}$$

lm – lumens
Y_λ – photopic response
S_λ – measured spectra
A_λ – Interpolated Assignment
T_λ – Test measurement
Cal_λ – Measurement of Standard

$u_i(\text{lumen}) = c_i \cdot u(A_i)$



$u_i(\text{color}) = c_i \cdot u(A_i)$



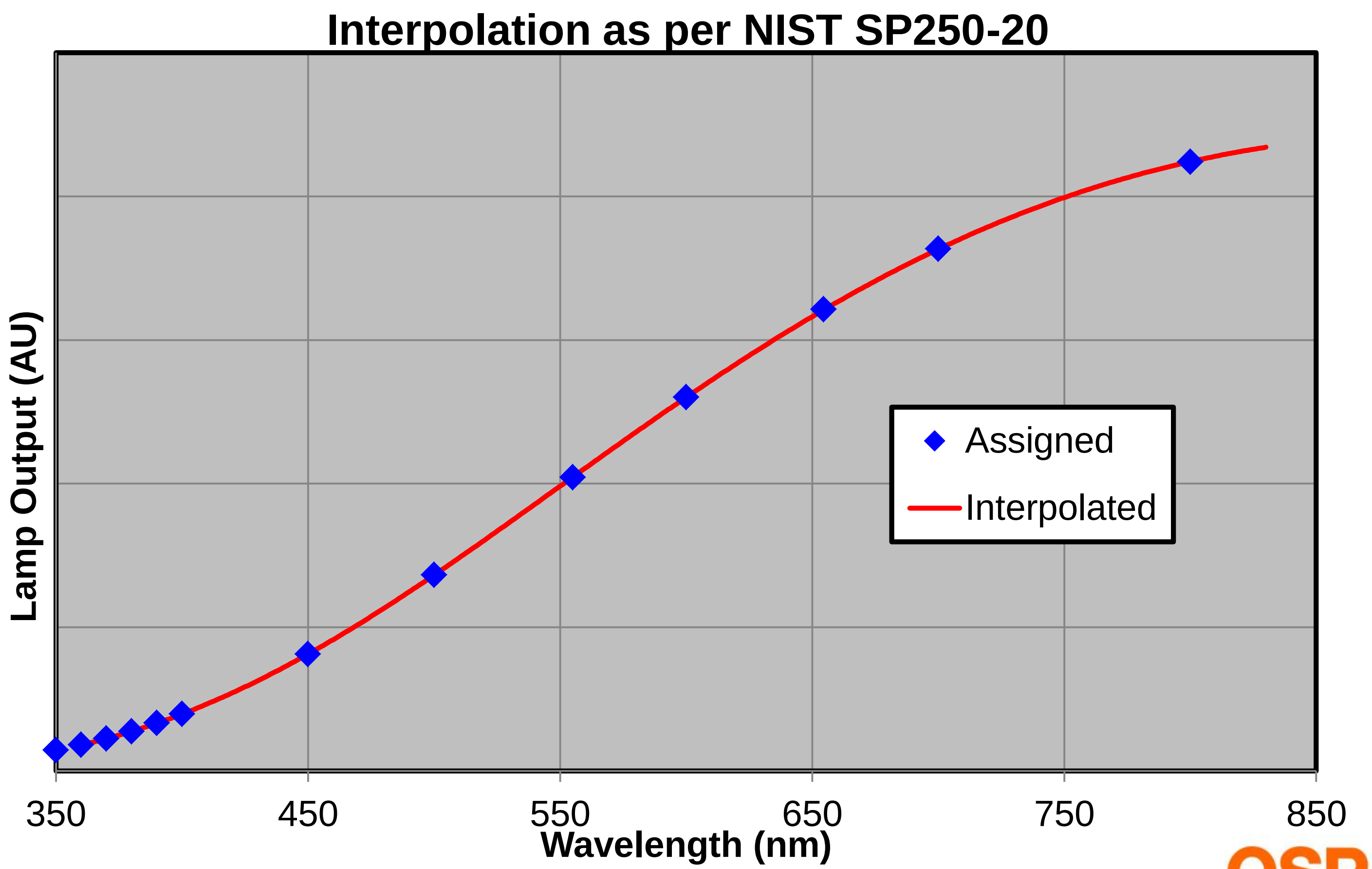
Uncertainty arising from the Standard - applied at each wavelength

u_c (lumen) = 0.034% ! $u_{555\text{ nm}}$ = 0.39 %

u_c (CIEx) = 0.00007 ?

u_c (CIEy) = 0.00010 ?

Second Method
apply uncertainty at interpolation



Full measurement process

$\text{Assignment}_\lambda = g(N_1, N_2, \ldots, N_{12})$ N_i = assignment at NIST wavelengths

λ = 360 nm to 830 nm

g is nonlinear least squares fit which I will not attempt to differentiate.

Apply uncertainty to each NIST assignment in turn and propagate through the measurement and color calculations.

Relate the differences in photometric values.

Apply uncertainty before interpolation

$$Lumen = f_{lm}(S_1, S_2, \ldots S_n)$$

$$CIEx = f_{CIEx}(S_1, S_2, \ldots S_n)$$

$$CIEy = f_{CIEy}(S_1, S_2, \ldots S_n)$$

$$u_i(Lumen) = c_i \bullet u(S_i)$$

$$u_i(Lumen) = c_i \bullet u(A_i)$$

Perfect measurement Δ%S = Δ%A

$$A = g(NIST)$$

$$u_i(Lumen) = c_i \bullet u(N_i)$$

N_i = Assigned value at NIST wavelength i

$u(a_i)$ is supplied

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Lumen Measurement Equation(s)

$$lm = 683 \bullet \begin{pmatrix} S_{360} \\ S_{361} \\ \dots \\ S_{830} \end{pmatrix}^T \begin{pmatrix} \bar{Y}_{360} \\ \bar{Y}_{361} \\ \dots \\ \bar{Y}_{830} \end{pmatrix}$$

$$\begin{pmatrix} S_{360} \\ S_{361} \\ \dots \\ S_{830} \end{pmatrix} = \begin{pmatrix} A_{360} \\ A_{361} \\ \dots \\ A_{830} \end{pmatrix} \bullet \begin{pmatrix} T_{360} \\ T_{361} \\ \dots \\ T_{830} \end{pmatrix} \bullet \begin{pmatrix} 1/Cal_{360} \\ 1/Cal_{361} \\ \dots \\ 1/Cal_{830} \end{pmatrix}$$

$$\begin{pmatrix} A_{360} \\ A_{361} \\ \dots \\ A_{830} \end{pmatrix} = g(\lambda)$$

$$g(\lambda) = \frac{e^{c1 + c2/\lambda + c3/\lambda^2 + c4/\lambda^3 + c5/\lambda^4}}{\lambda^5}$$

$$\begin{pmatrix} c1 \\ c2 \\ c3 \\ c4 \\ c5 \end{pmatrix} = h \begin{pmatrix} N_{360} \\ N_{370} \\ \dots \\ N_{800} \end{pmatrix}$$

h(λ) – Polynomial Least Square Fit accomplished without iteration

lm – lumens
Y_λ – photopic response
S_λ – measured spectra
A_λ – Interpolated Assignment
T_λ – Test measurement
Cal_λ – Measurement of Standard
$g(\lambda)$ – Wien's approximation
N_λ – NIST Assignment

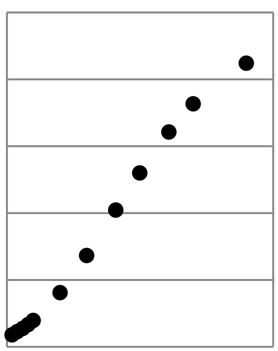
OSRAM

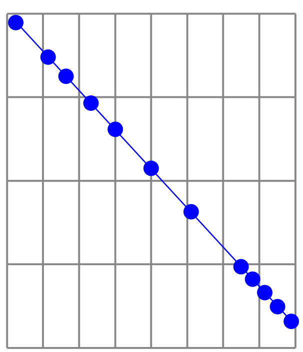
Measurement Equation in EXCEL

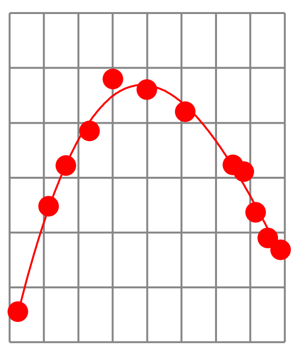
x	**0.00004**	0.334	0.334
y	**0.00007**	0.334	0.334
lm	**0.10%**	5583	5588

Linear Fit		3rd Order Fit	
	44.66		**0.9921**
	-4.69		**0.0100**
			-0.004
			0.0004

NIST Values nm	E	u	E	Wien's Approximation			
360	9.023	0.62%	9.02	2.78	31.63	31.64	0.9997
370	11.20	0.60%	11.20	2.70	31.98	31.99	0.9998
380	13.72	0.58%	13.72	2.63	32.32	32.32	0.9999
390	16.61	0.56%	16.61	2.56	32.64	32.64	1.0000
400	19.78	0.55%	19.78	2.50	32.94	32.94	1.0000
450	40.62	0.46%	40.62	2.22	34.25	34.24	1.0002
500	68.17	0.42%	68.17	2.00	35.30	35.28	1.0003
555	**102.2**	0.39%	**102.6**	1.80	36.23	36.21	1.0004
600	130.0	0.37%	130.0	1.67	36.85	36.85	1.0002
654.6	160.6	0.35%	160.6	1.53	37.50	37.50	1.0000
700	181.7	0.34%	181.7	1.43	37.96	37.96	0.9999
800	212.0	0.31%	212.0	1.25	38.78	38.80	0.9995

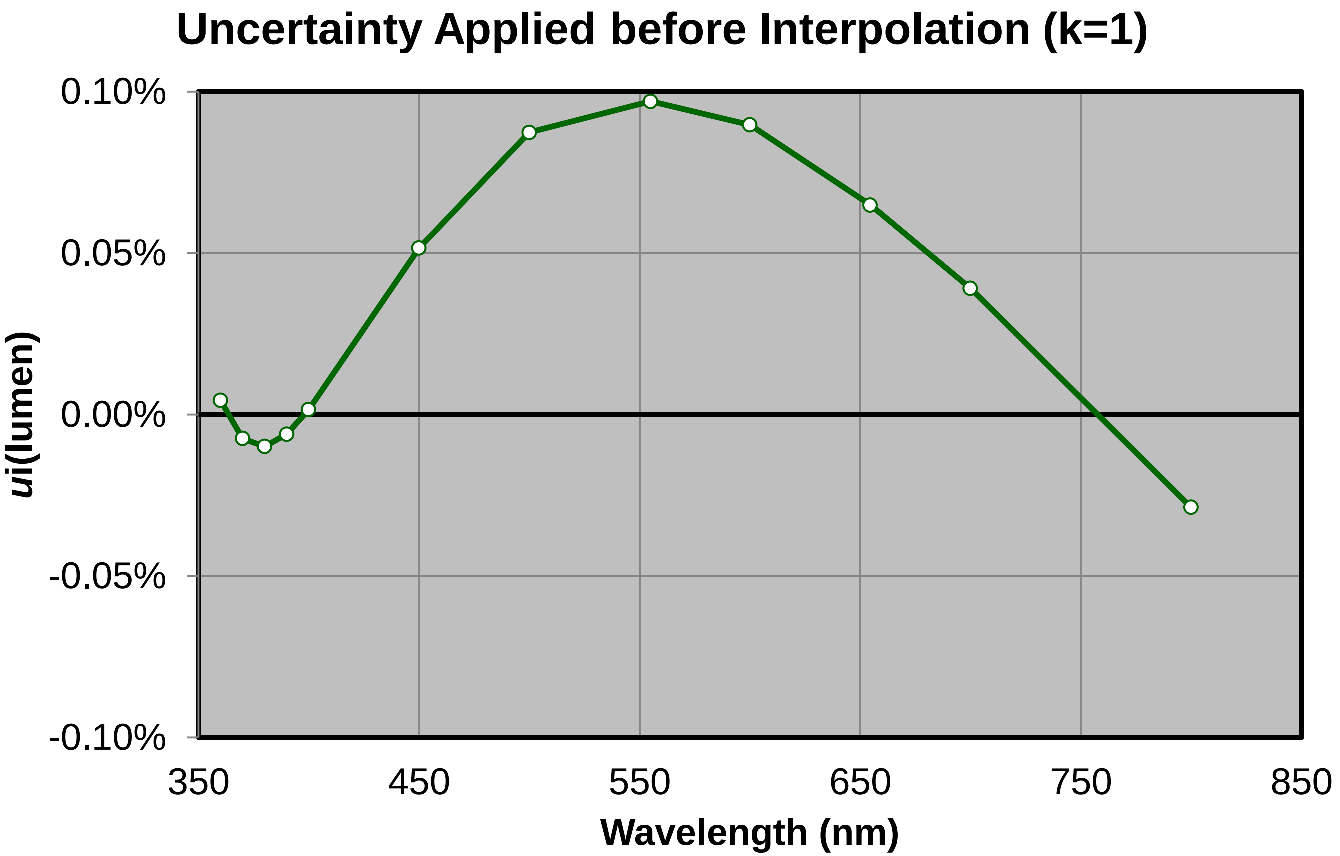




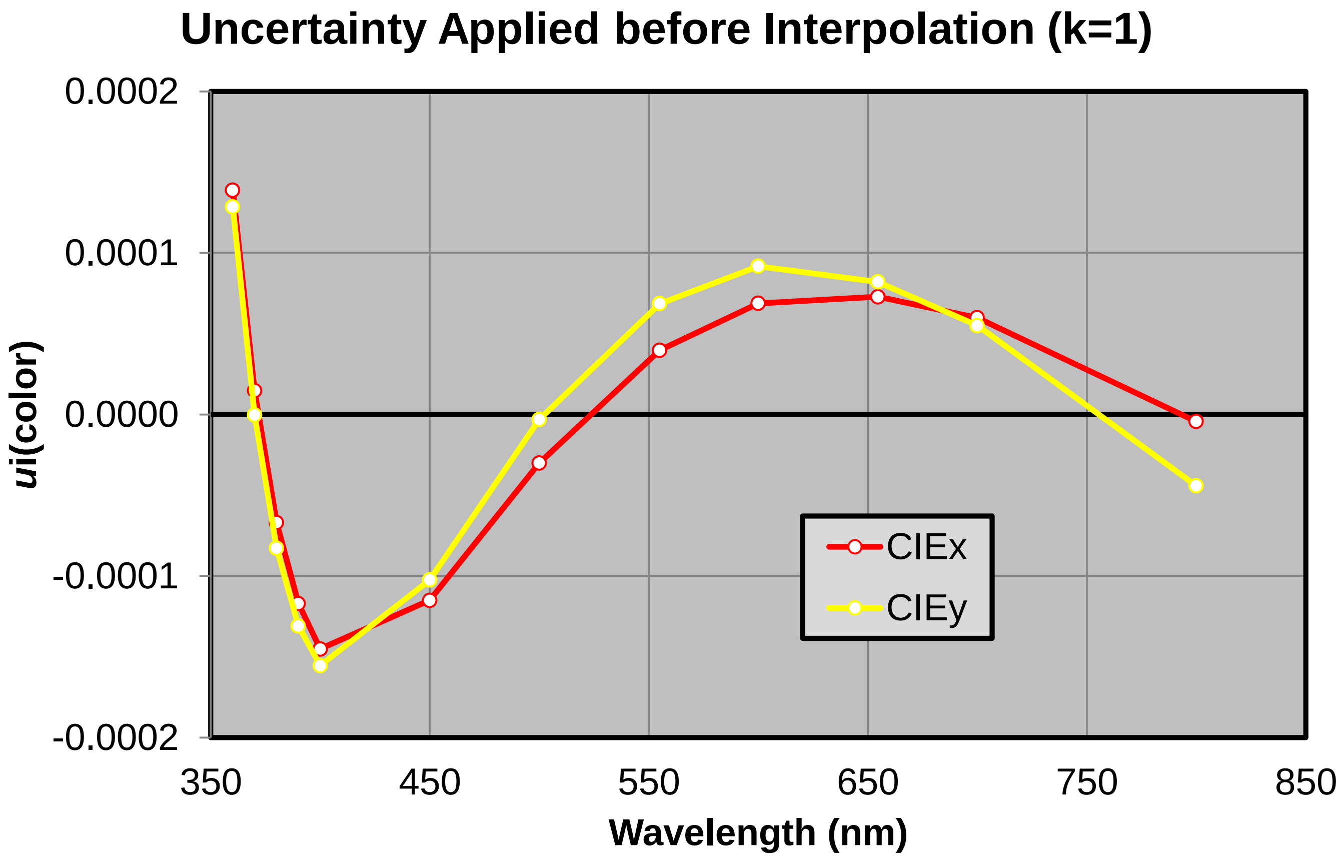


Interpolated nm	Perturbed	Baseline	Test Source nm	E	E	Tristimulus Functions nm	X	Y	Z
360	9.02	9.02	360	0.000	0.000	360	0.000	0.000	0.001
361	9.22	9.22	361	0.000	0.000	361	0.000	0.000	0.001
362	9.43	9.43	362	0.000	0.000	362	0.000	0.000	0.001
363	9.64	9.64	363	0.000	0.000	363	0.000	0.000	0.001
364	9.86	9.86	364	0.000	0.000	364	0.000	0.000	0.001
365	10.08	10.08	365	0.000	0.000	365	0.000	0.000	0.001
366	10.30	10.30	366	0.000	0.000	366	0.000	0.000	0.001
367	10.52	10.52	367	0.000	0.000	367	0.000	0.000	0.001
368	10.75	10.75	368	0.000	0.000	368	0.000	0.000	0.002
369	10.98	10.98	369	0.000	0.000	369	0.000	0.000	0.002
370	11.21	11.21	370	0.000	0.000	370	0.000	0.000	0.002
371	11.45	11.45	371	0.000	0.000	371	0.000	0.000	0.002
372	11.69	11.69	372	0.000	0.000	372	0.001	0.000	0.002
373	11.93	11.93	373	0.000	0.000	373	0.001	0.000	0.003
374	12.18	12.18	374	0.000	0.000	374	0.001	0.000	0.003
375	12.43	12.43	375	0.000	0.000	375	0.001	0.000	0.003
376	12.68	12.68	376	0.000	0.000	376	0.001	0.000	0.004
377	12.94	12.94	377	0.000	0.000	377	0.001	0.000	0.005
378	13.20	13.20	378	0.000	0.000	378	0.001	0.000	0.005
379	13.46	13.46	379	0.000	0.000	379	0.001	0.000	0.006
380	13.73	13.73	380	0.000	0.000	380	0.001	0.000	0.006
381	14.00	14.00	381	0.000	0.000	381	0.002	0.000	0.007
382	14.27	14.27	382	0.000	0.000	382	0.002	0.000	0.008
383	14.55	14.55	383	0.000	0.000	383	0.002	0.000	0.009
384	14.83	14.83	384	0.000	0.000	384	0.002	0.000	0.009

u_i(lumen) = $c_i \cdot u(N_i)$



$u_i(\text{color}) = c_i \cdot u(N_i)$



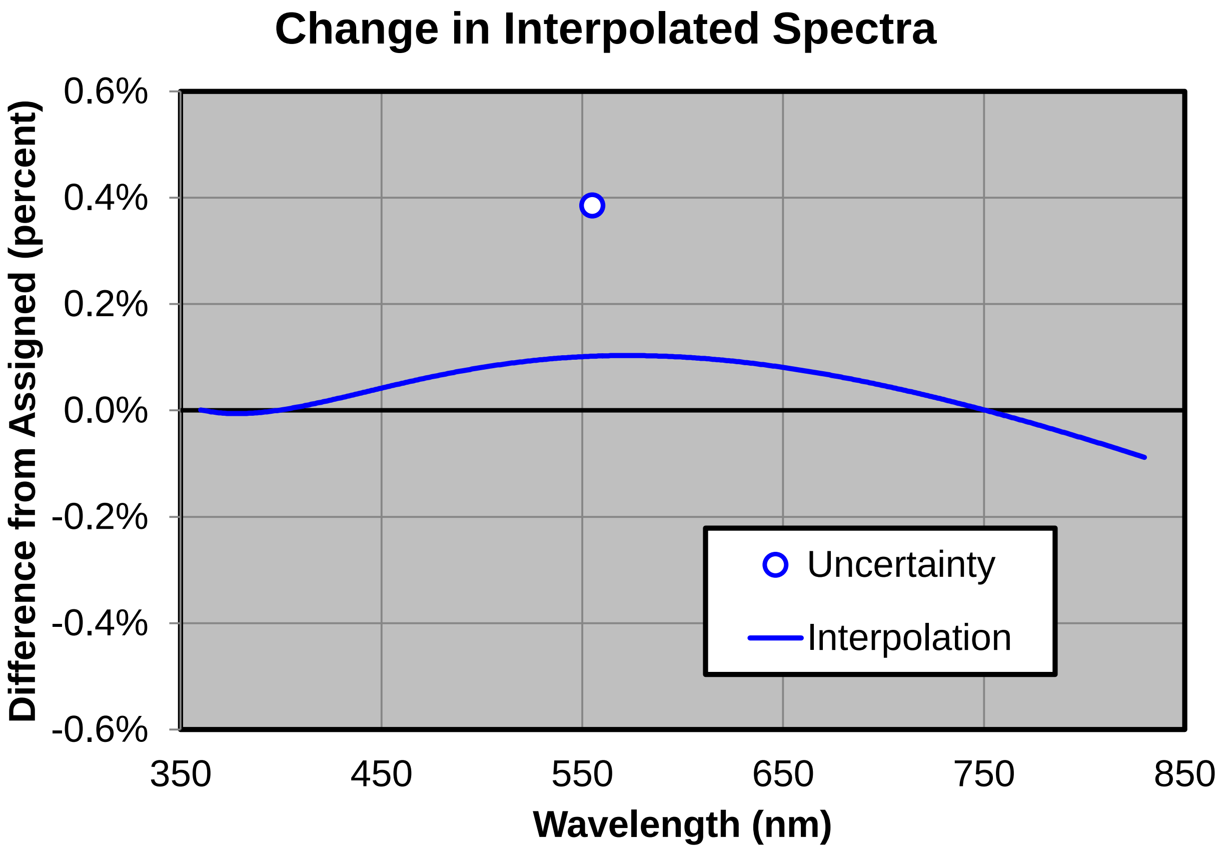
Combined Uncertainty arising from the Standard - applied before interpolation

u_c (lumen) = 0.19% X u_{555nm} = 0.39 %

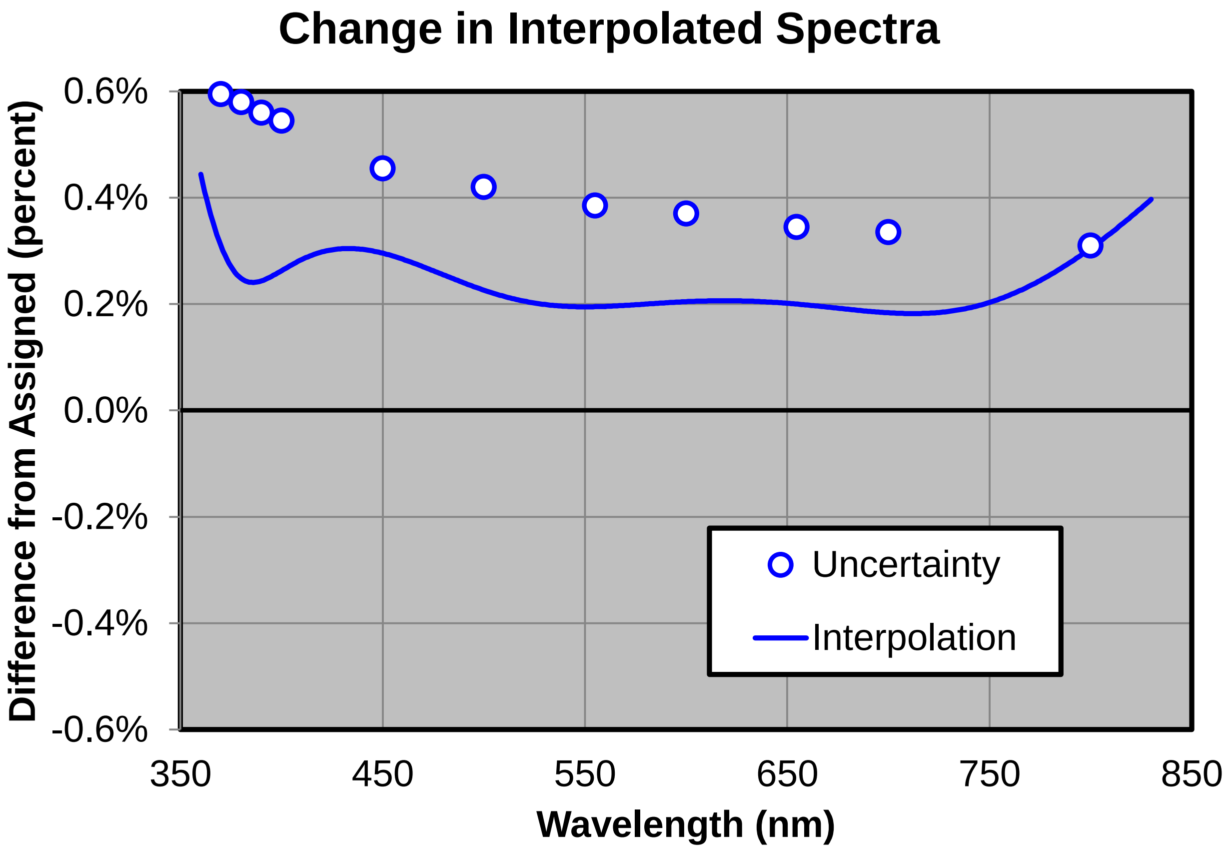
u_c (CIEx) = 0.00030 ?

u_c (CIEy) = 0.00032 ?

Assigned value of one wavelength changed



Assigned value of all wavelengths changed



Sources of Uncertainty	250	350	450	555	655	900	1600	2000	2300	2400	
1) HTBB Temperature Uncertainty (0.43 K at 2950 K)	0.28	0.20	0.16	0.13	0.11	0.08	0.05	0.04	0.03	0.03	0.42 K
2) HTBB spectral emissivity	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	
3) HTBB spatial uniformity	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	
4) HTBB temporal stability (0.05 K / hour)	0.03	0.02	0.02	0.02	0.01	0.01	0.01	0.00	0.00	0.00	0.04 K
5) Geometric Factors in Irradiance Transfer	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	
6) Spectroradiometer Responsivity Stability	0.30	0.30	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.50	
7) Wavelength accuracy (0.05 nm)	0.29	0.13	0.06	0.04	0.02	0.000	0.010	0.010	0.010	0.010	
8) Lamp / Spectroradiometer transfer	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	0.05	
9) Lamp Current Stability	0.03	0.02	0.02	0.02	0.02	0.01	0.01	0.01	0.01	0.01	0.08 K
10) Uncertainty of the primary working standards	**0.52**	**0.40**	**0.25**	**0.23**	**0.21**	**0.20**	**0.19**	**0.18**	**0.18**	**0.51**	
11) lamp-to-lamp transfer	0.25	0.15	0.10	0.10	0.10	0.10	0.10	0.15	0.15	0.20	
12) Long-term stability of working standards	0.66	0.47	0.36	0.30	0.25	0.18	0.10	0.08	0.07	0.07	0.99 K
13) Uncertainty of the issued	**0.87**	**0.63**	**0.45**	**0.39**	**0.34**	**0.29**	**0.24**	**0.25**	**0.25**	**0.55**	
uncorrelated	0.50	0.37	0.21	0.21	0.21	0.21	0.21	0.23	0.23	0.55	
correlated	0.72	0.51	0.40	0.32	0.27	0.20	0.12	0.10	0.09	0.09	1.08 K

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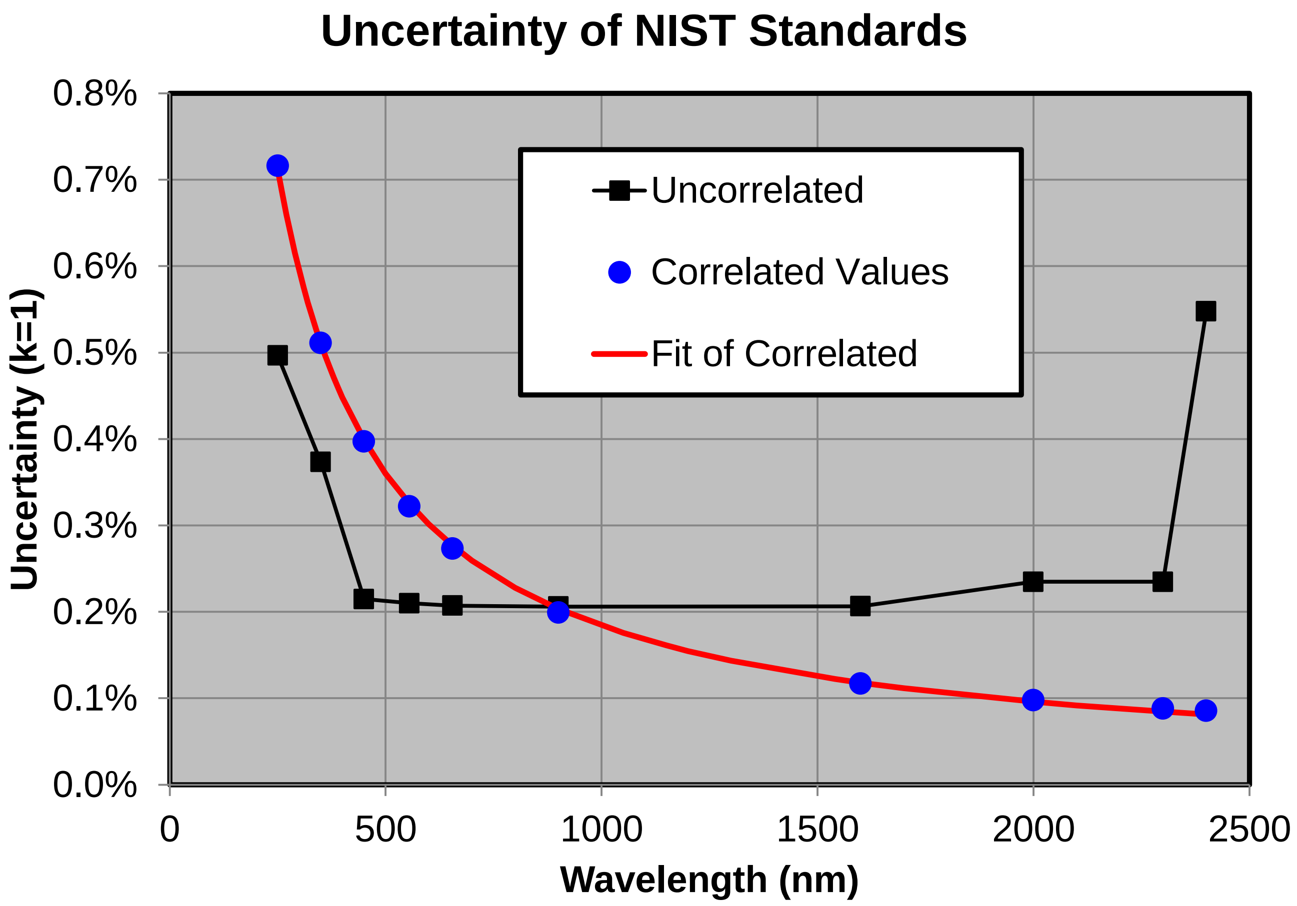
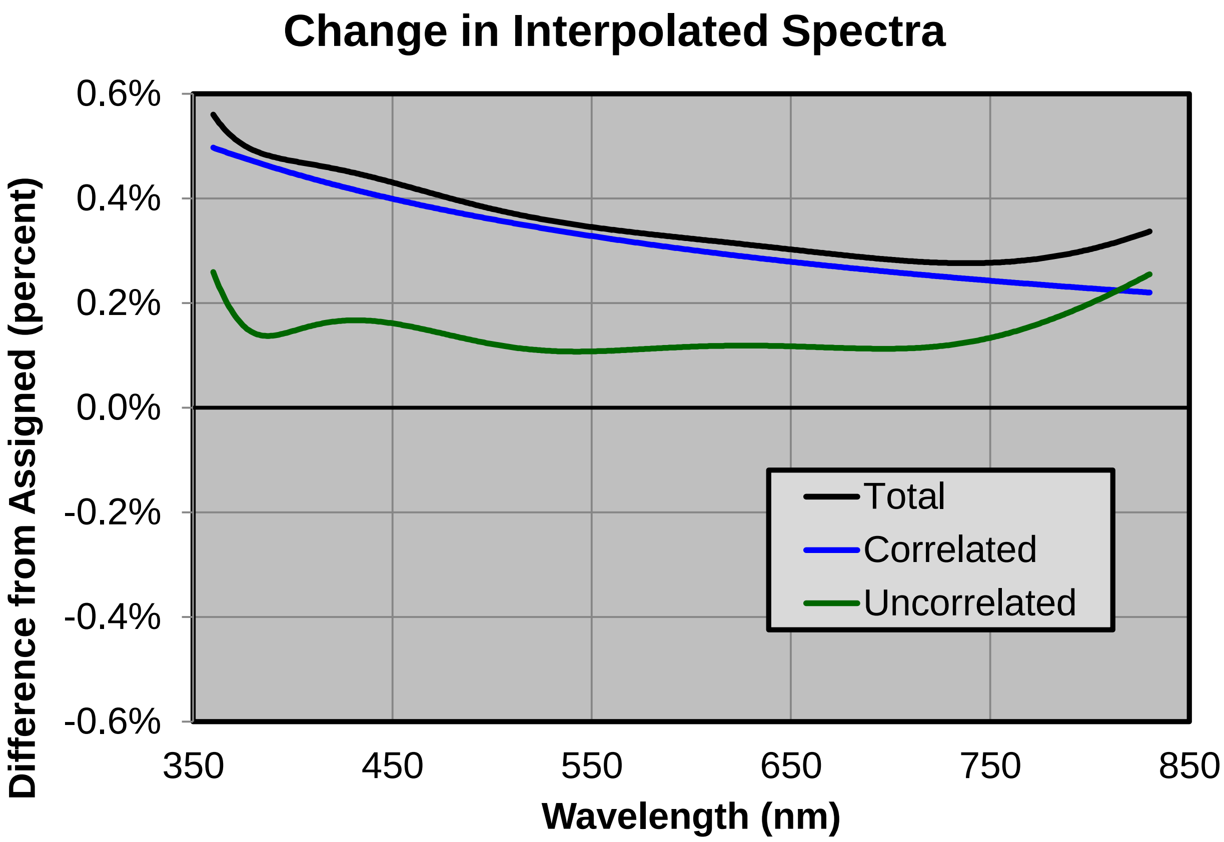
Separation of correlated and uncorrelated



Correlation's impact on Interpolation



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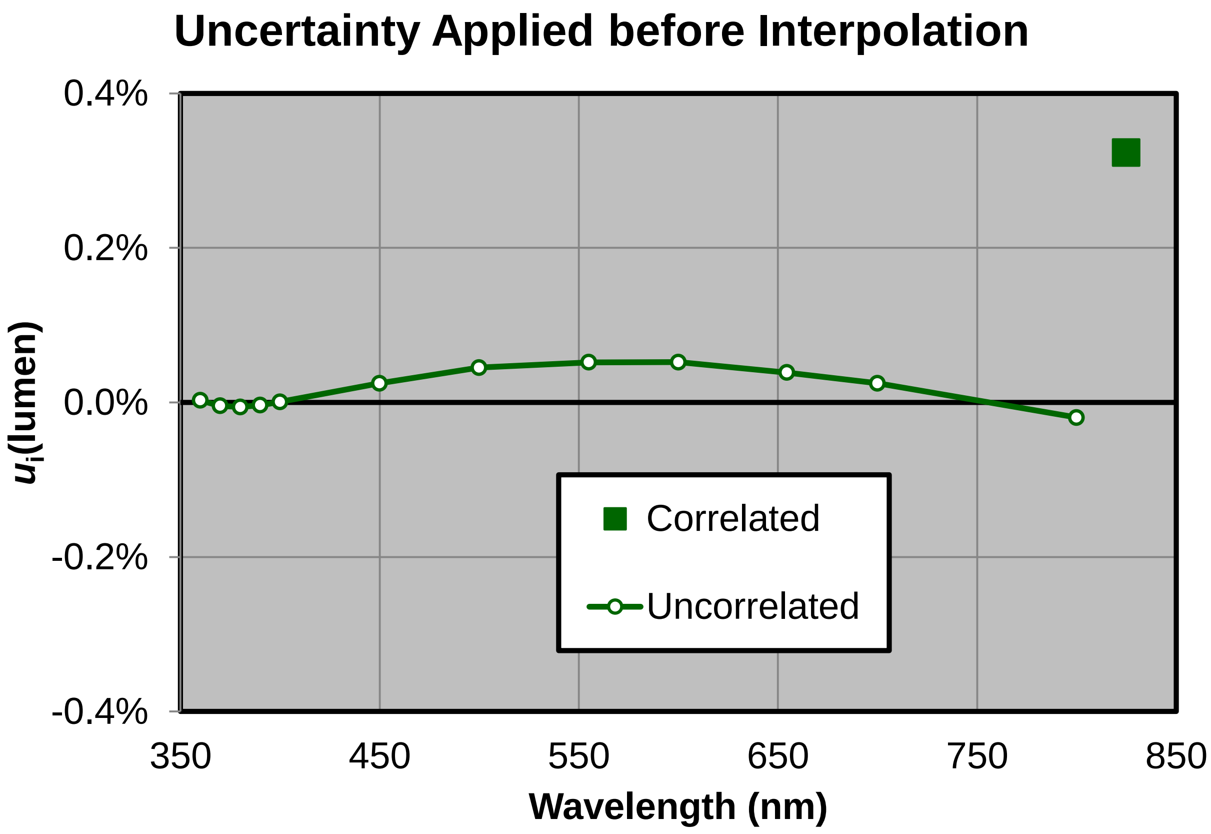
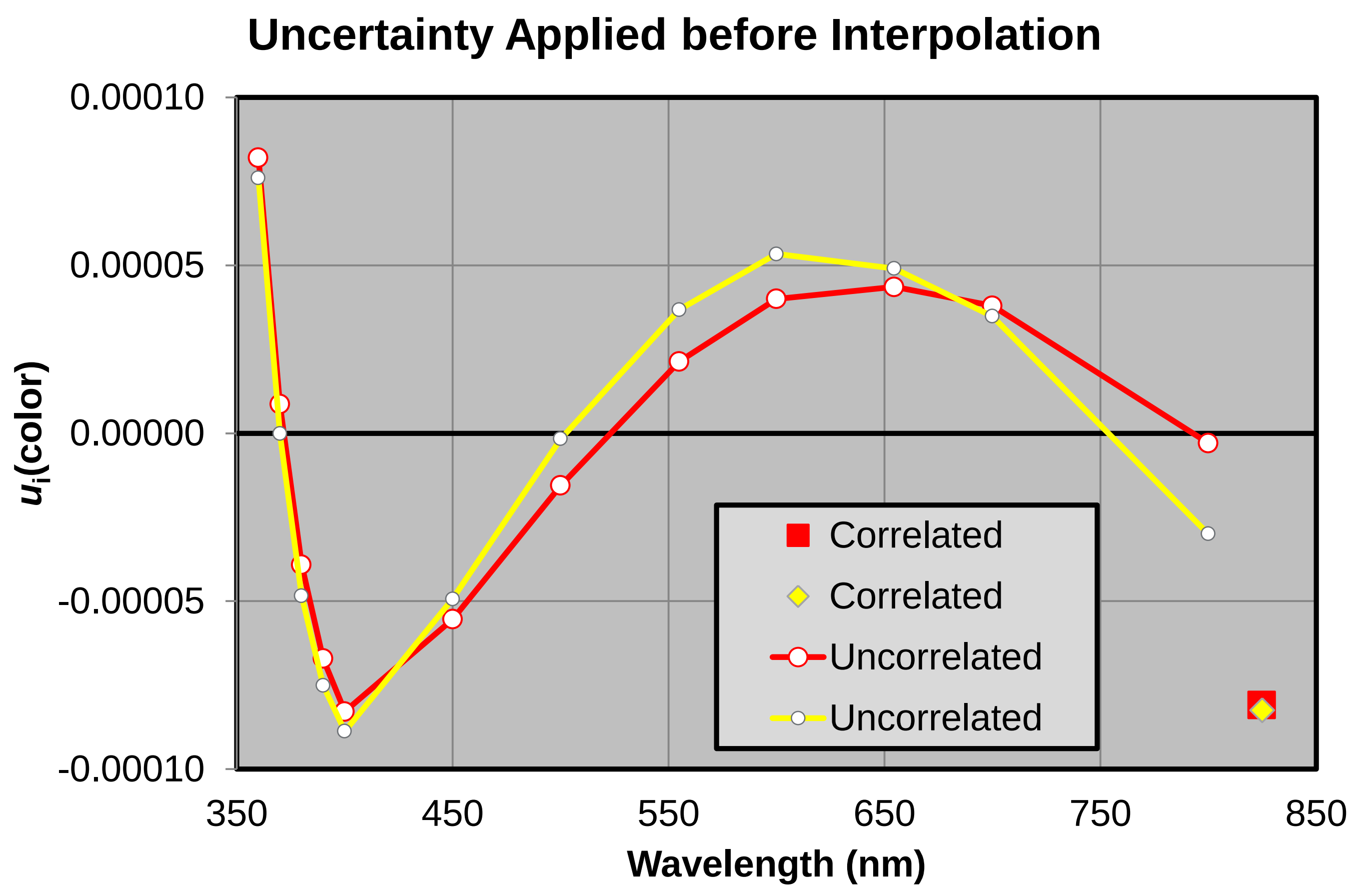
Uncertainty Components Separated



Uncertainty Components Separated



Uncertainty arising from the Standard - Correlations applied before interpolation

u_c (lumen) = 0.34% √ u_{555nm} = 0.39 %

u_c (CIEx) = 0.00019 ?

u_c (CIEy) = 0.00020 ?

correlation lowered the uncertainties

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Monte Carlo Method

$$lm = f(N_{350}, N_{360}, \ldots N_{800})$$

$$u_{350}(lm) = f(N_{350} \bullet (1 + u(N_{350})), N_{360}, \ldots N_{800})$$

$$u_C(lm) = \sqrt{\sum u_i^{\,2}(lm)}$$

$$lm = f(N_{350} \bullet R_{350}, N_{360} \bullet R_{360}, \ldots N_{800} \bullet R_{800})$$

$$R_i = 1 + u(N_i) \bullet Random_{NORMAL}$$

lm is then a Problity Distribution Function

$u_C(lm)$ is the standard deviation of lm

Monte Carlo Simulation
- Correlations applied before interpolation

u_c (lumen) = 0.35%

u_c (CIEx) = 0.00019

u_c (CIEy) = 0.00020

Agrees with discreet analysis

Color Uncertainty Relationship