# Uncertainty analysis for NIST spectral irradiance measurements

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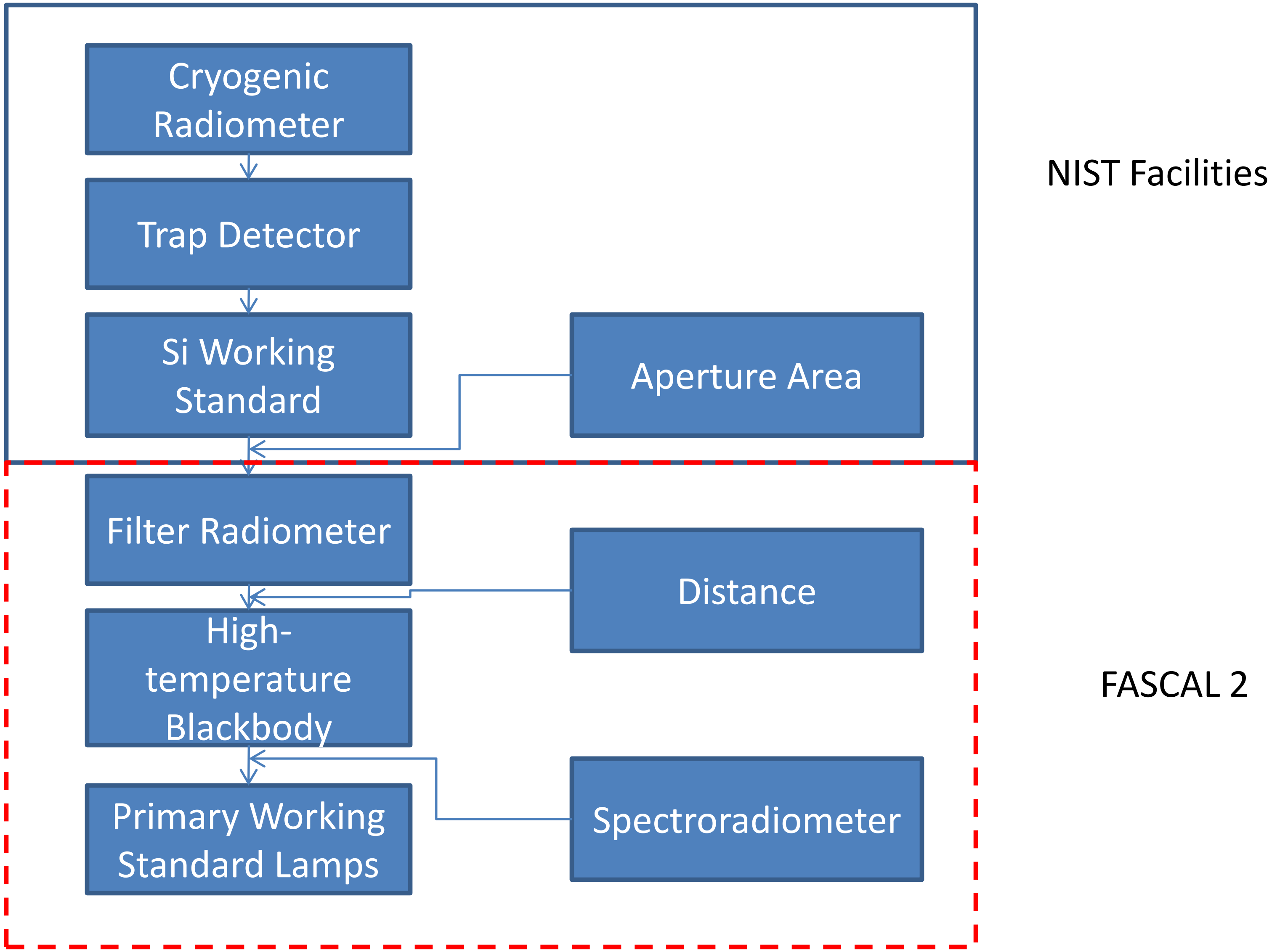
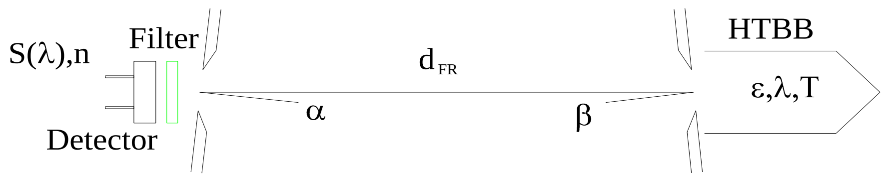
# Outline

1. NIST uncertainties
   A. Physical basis of the uncertainties (Scale realization)
   B. Evaluation of the uncertainties
2. User uncertainties
   A. Photometric uses (CCT, lumen realization)
   B. Spectroradiometric uses (interpolations)

# Traceability Chain

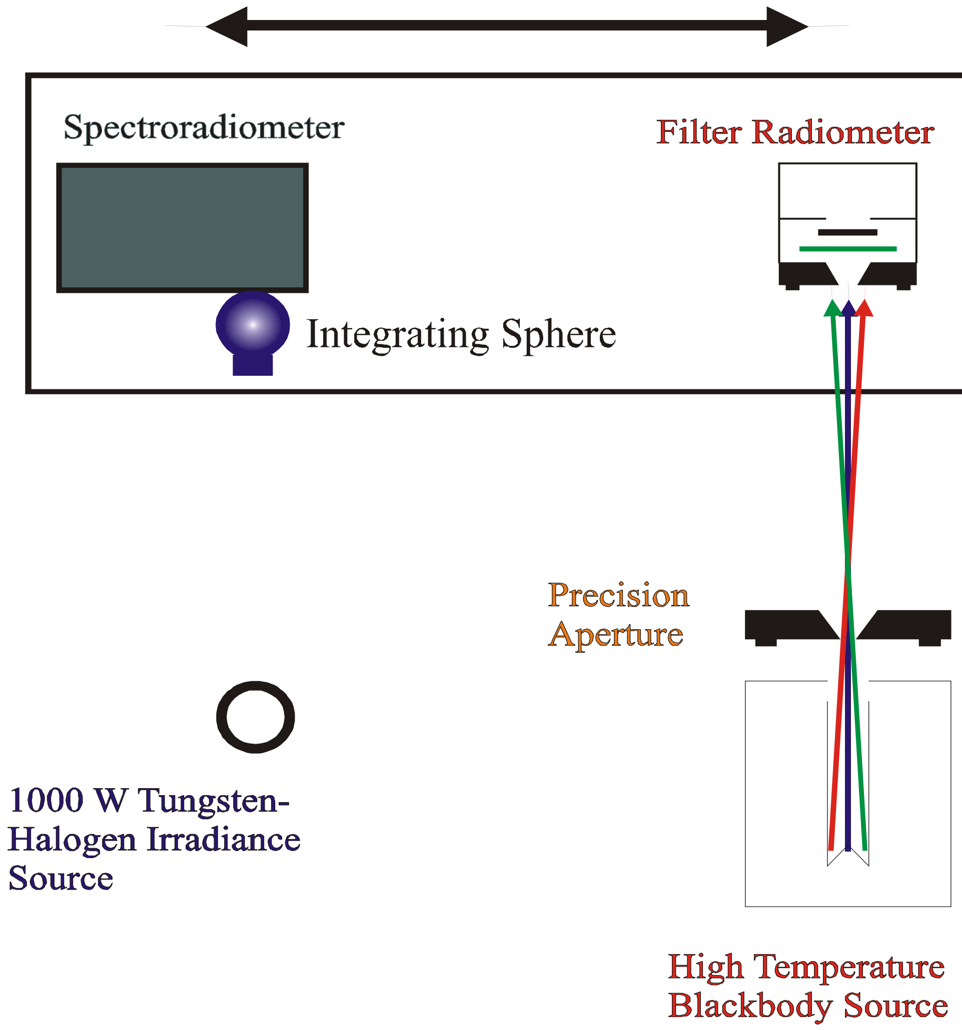


# Setup and measurement equation for detector-based temperatures

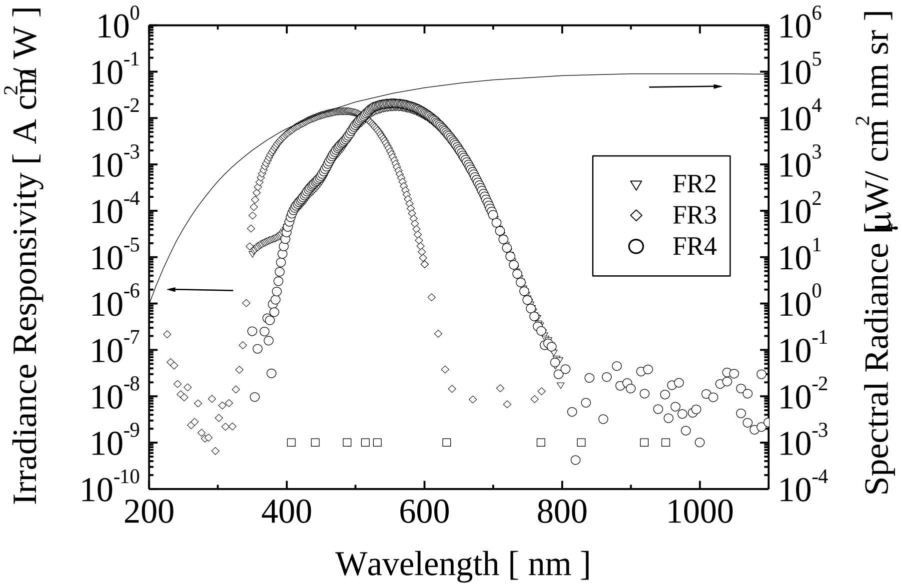



$$v = G\,\frac{A_{\mathrm{HTBB}}\cos(\beta)\;A_{\mathrm{FR}}\cos(\alpha)}{d_{\mathrm{FR}}^{2}}\int S(\lambda)\cdot\varepsilon\,\frac{c_{1L}}{n^{2}\lambda^{5}}\,\frac{1}{\exp\left(c_{2}/(n\lambda T)\right)-1}\,d\lambda$$

# Spectral irradiance scale realization



# Filter Radiometer Uncertainties

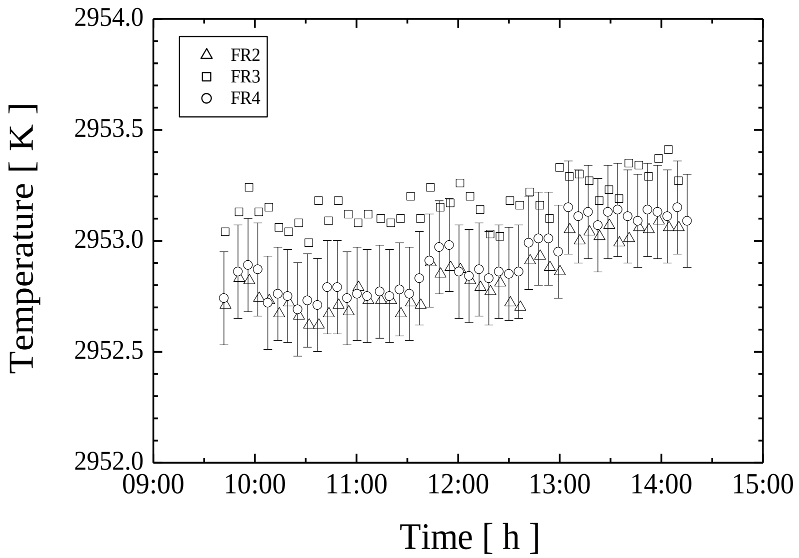


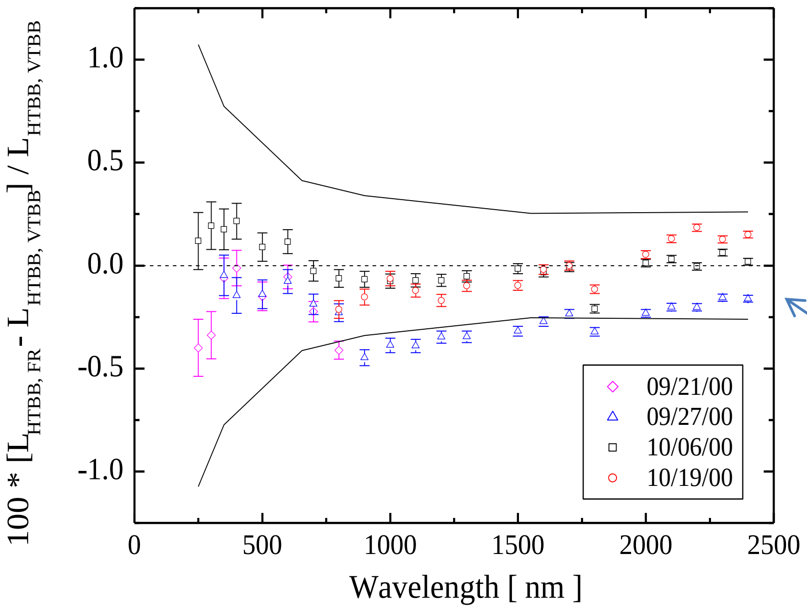

0.22 % (k=2) uncertainties in S(λ)

$$v = G\,\frac{A_{HTBB}\cos(\beta)\;A_{FR}\cos(\alpha)}{d_{FR}^{2}} \int S(\lambda)\cdot \varepsilon\,\frac{c_{1L}}{n^{2}\lambda^{5}}\,\frac{1}{\exp\left(c_{2}/(n\lambda T)\right)-1}\,d\lambda$$

# HTBB characterizations

Stability of 0.1 K/h



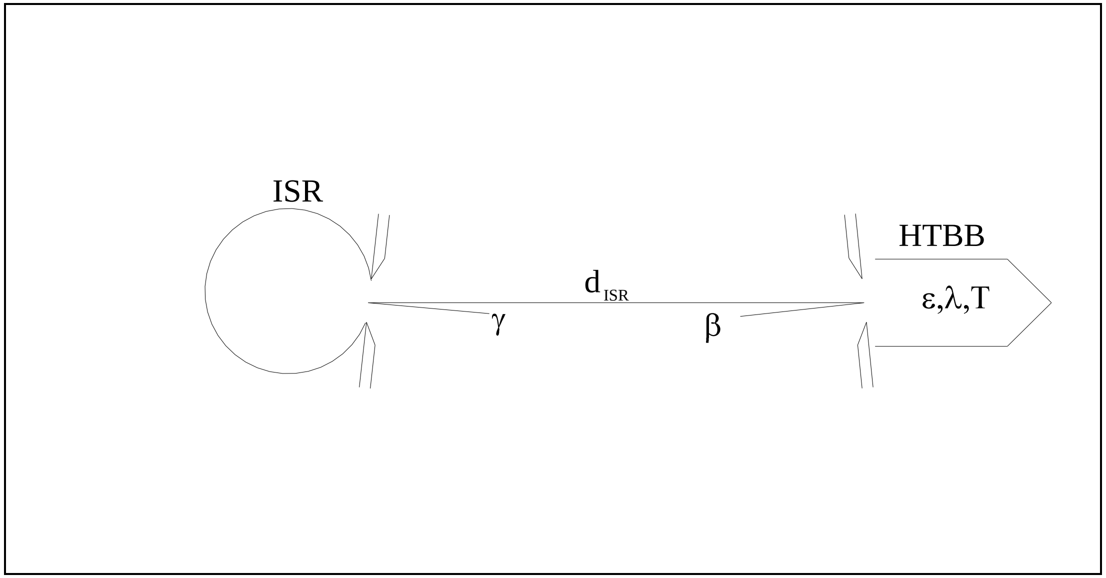

Emissivity = 0.99

# Uncertainty Budget for Spectral Irradiance Calibrations

| Source of Uncertainty | Relative Expanded Uncertainties ($k$ = 2) [%] | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 250 | 350 | 450 | 555 | 654.6 | 900 | 1600 | 2000 | 2300 | 2400 |
| 1. HTBB temperature uncertainty (0.86 K at 2950 K) (B) | 0.57 | 0.41 | 0.32 | 0.26 | 0.22 | 0.16 | 0.09 | 0.08 | 0.07 | 0.07 |
| 2. HTBB spectral emissivity (B) | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 |
| 3. HTBB spatial uniformity (B) | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 |
| 4. HTBB temporal stability (0.1 K / h) (B) | 0.07 | 0.05 | 0.04 | 0.03 | 0.03 | 0.02 | 0.01 | 0.01 | 0.01 | 0.01 |
| 5. Geometric factors in irradiance transfer (B) | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 |
| 6. Spectroradiometer responsivity stability (B) | 0.60 | 0.60 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 1.00 |
| 7. Wavelength uncertainty (0.1 nm) (B) | 0.58 | 0.26 | 0.13 | 0.07 | 0.04 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 |
| 8. Lamp/spectroradiometer transfer (B) | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 |
| 9. Lamp current stability (B) | 0.07 | 0.05 | 0.04 | 0.03 | 0.03 | 0.02 | 0.02 | 0.01 | 0.01 | 0.01 |
| **Total uncertainty of the primary working standards** | **1.03** | **0.80** | **0.50** | **0.45** | **0.43** | **0.40** | **0.37** | **0.37** | **0.37** | **1.02** |
| 10. Lamp-to-lamp transfer (A) | 0.50 | 0.30 | 0.20 | 0.20 | 0.20 | 0.20 | 0.20 | 0.30 | 0.30 | 0.40 |
| 11. Long-term stability of primary working standards (B) | 1.31 | 0.94 | 0.73 | 0.59 | 0.50 | 0.36 | 0.20 | 0.16 | 0.14 | 0.14 |
| **Overall uncertainty of the test with respect to SI units** | **1.74** | **1.27** | **0.91** | **0.77** | **0.69** | **0.57** | **0.47** | **0.50** | **0.49** | **1.11** |

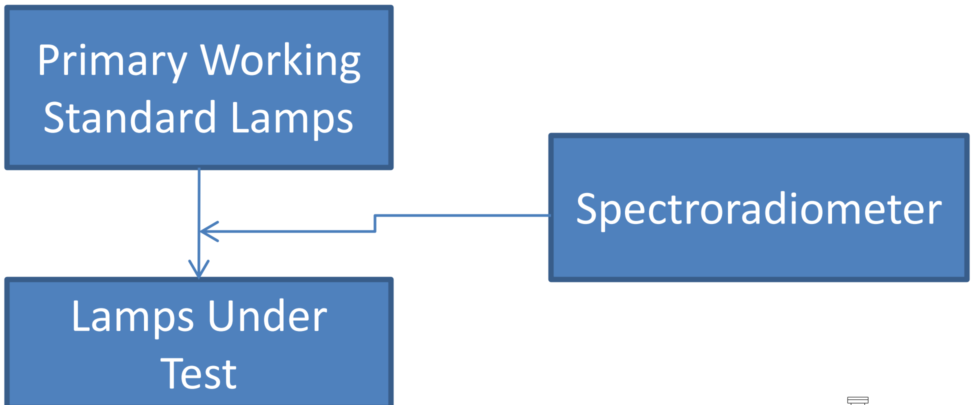
**Note:** The Type A or Type B evaluation of the uncertainty is indicated in parentheses.

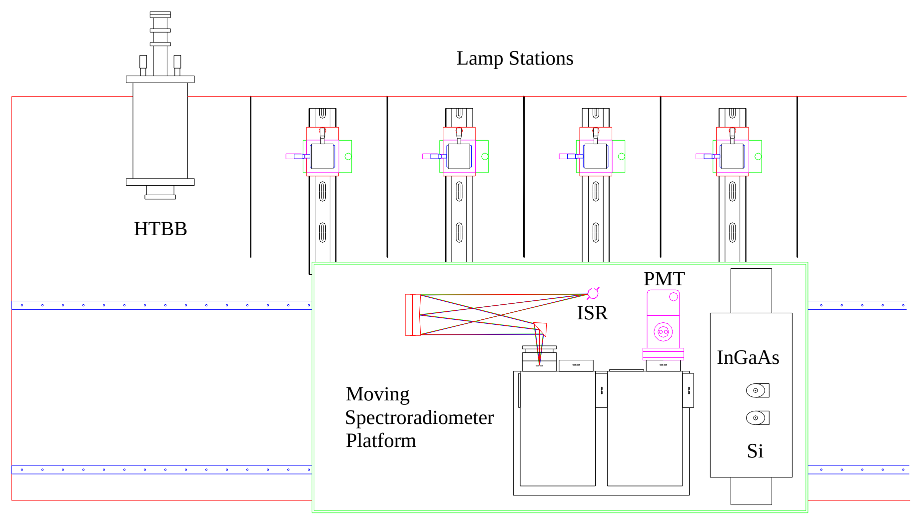
# HTBB to spectral irradiance responsivity




$$E_{HTBB}(\lambda) = \frac{A_{HTBB}\cos(\beta)\cos(\gamma)}{d_{ISR}^2}\,\varepsilon\,\frac{c_{1L}}{n^2\lambda^5}\,\frac{1}{\exp(c_2/(n\lambda T))-1}$$

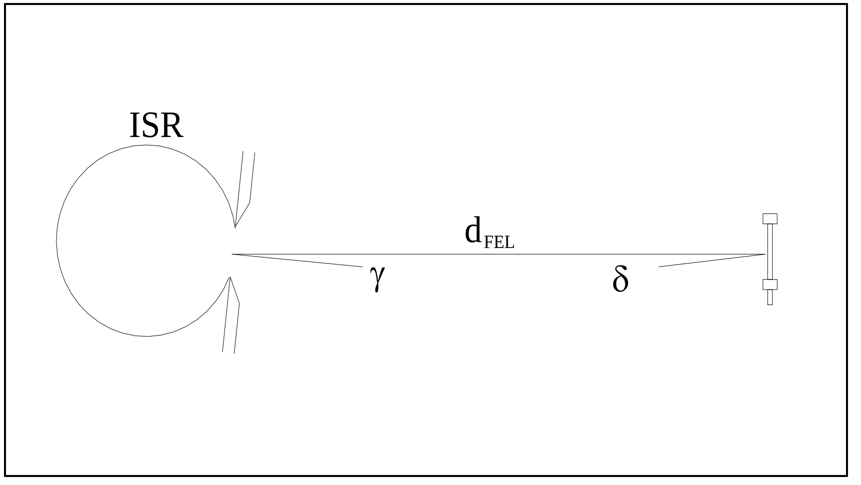
# Lamp-to-lamp transfer



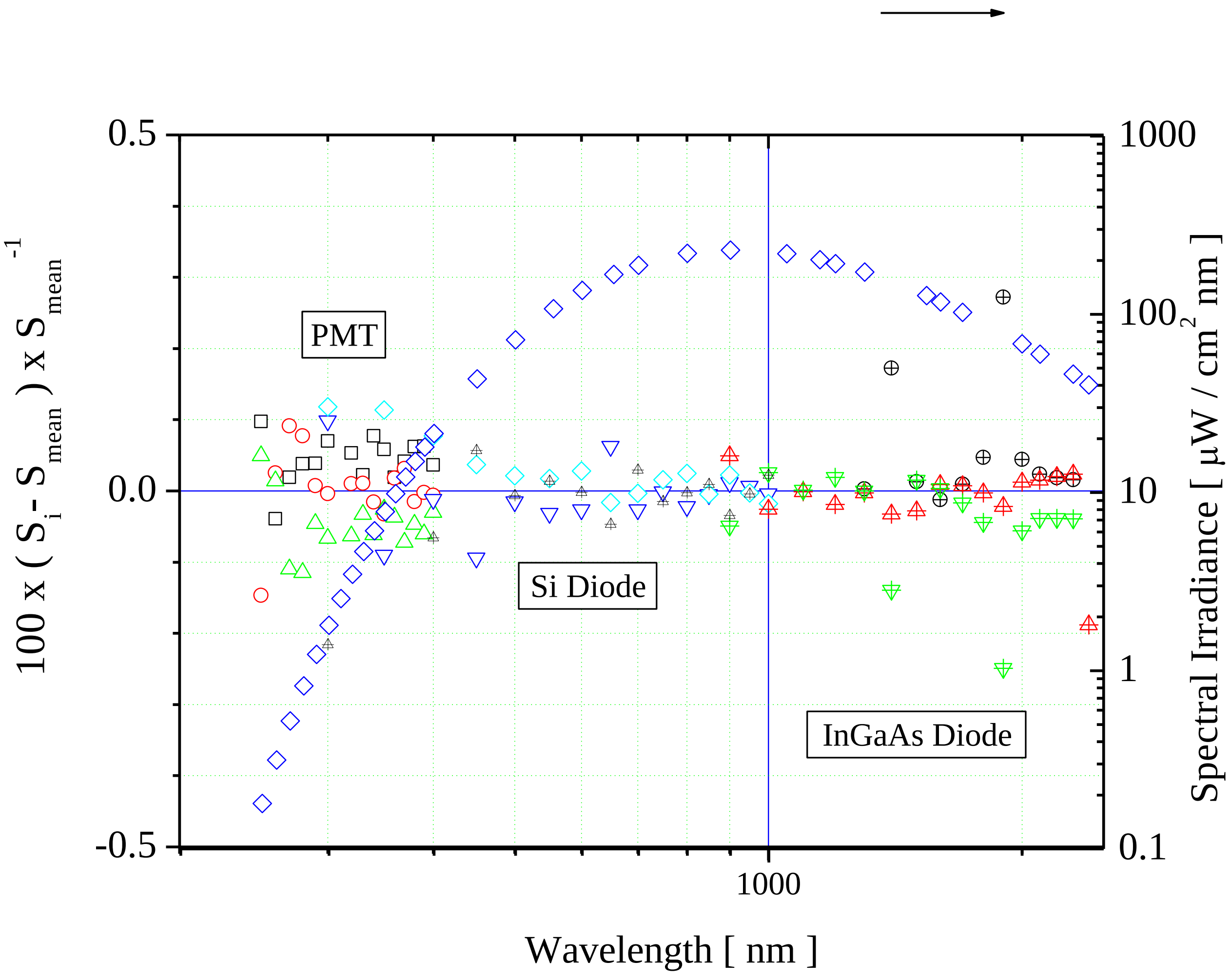

$$E_U(\lambda) = \frac{E_C(\lambda)}{S_C} S_U = R_C S_U$$

# Lamp-to-lamp transfer setup

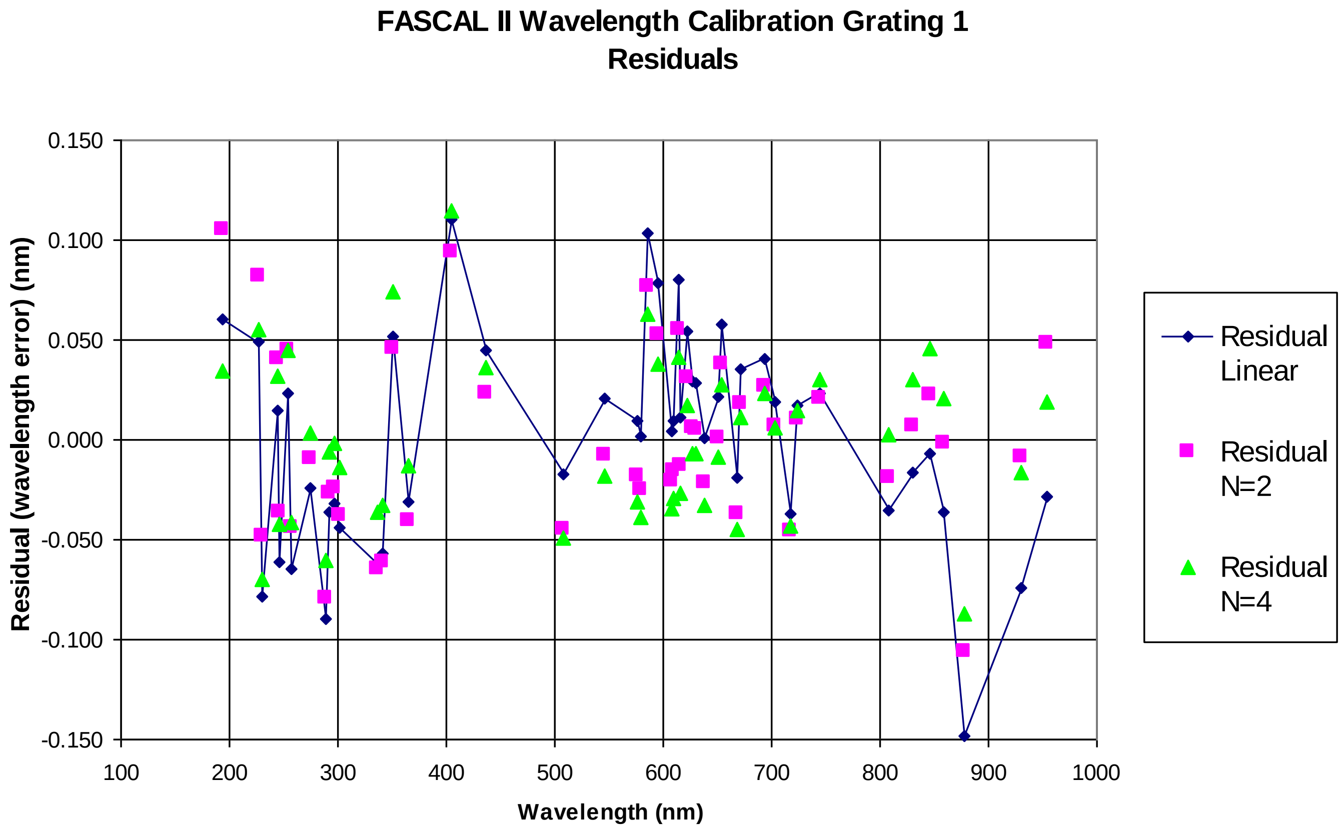



$$E_U(\lambda) = \frac{E_C(\lambda)}{S_C} S_U = R_C S_U$$

# Temporal stability of the transfer

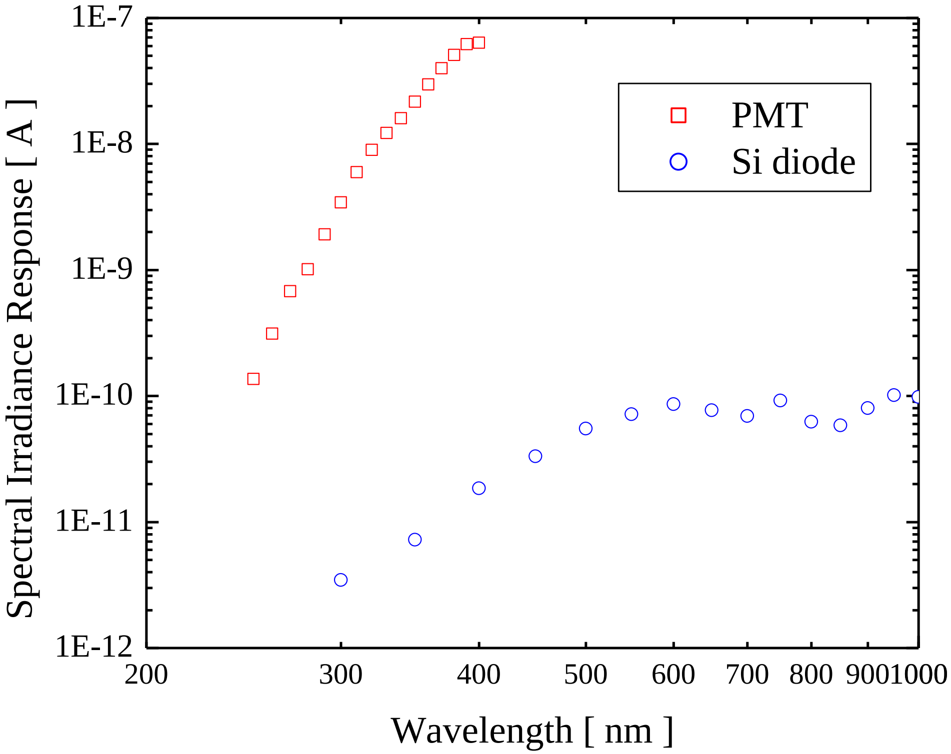


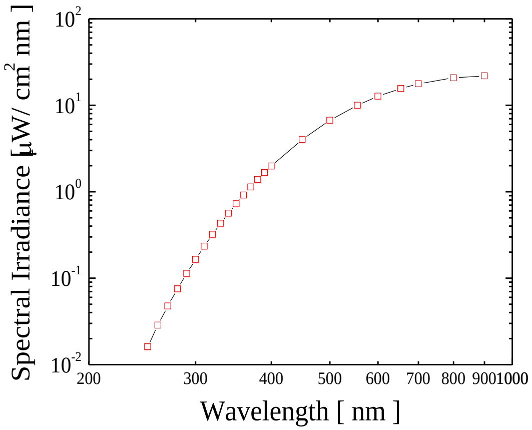
# Wavelength uncertainty




Measured using absolute encoder on the drive shaft of the sine bar screw

# Wavelength accuracy




The uncertainty due to wavelength calibration is proportional to the slope of the spectral response of the spectroradiometer.

If the spectral shape of the sources are known, then choose gratings which flatten the spectral response.

$$\frac{dI_E}{d\lambda} = m, \text{or } dI_E = m \cdot d\lambda$$

# Wavelength correlated uncertainty budget

| Source of Uncertainty | Relative Expanded Uncertainties (*k* = 2) [%] | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | 250 | 350 | 450 | 555 | 654.6 | 900 | 1600 | 2000 | 2300 | 2400 |
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| 2. HTBB spectral emissivity (B) Corr. | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 |
| 3. HTBB spatial uniformity (B) Corr. | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 |
| 4. HTBB temporal stability (0.1 K / h) (B) Corr. | 0.07 | 0.05 | 0.04 | 0.03 | 0.03 | 0.02 | 0.01 | 0.01 | 0.01 | 0.01 |
| 5. Geometric factors in irradiance transfer (B) Corr. | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 |
| 6. Spectroradiometer responsivity stability (B) Uncorr. | 0.60 | 0.60 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 0.30 | 1.00 |
| 7. Wavelength uncertainty (0.1 nm) (B) Uncorr. | 0.58 | 0.26 | 0.13 | 0.07 | 0.04 | 0.01 | 0.01 | 0.01 | 0.01 | 0.01 |
| 8. Lamp/spectroradiometer transfer (B) Uncorr. | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 | 0.10 |
| 9. Lamp current stability (B) Corr. | 0.07 | 0.05 | 0.04 | 0.03 | 0.03 | 0.02 | 0.02 | 0.01 | 0.01 | 0.01 |
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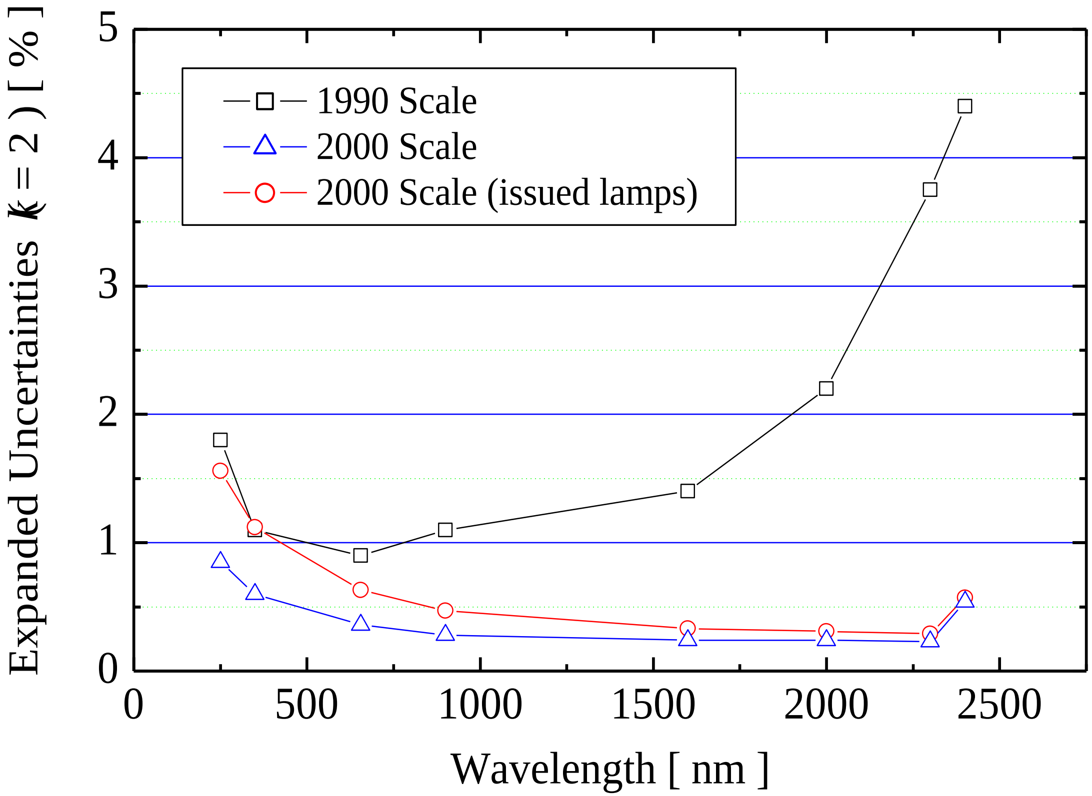
| **Overall uncertainty of the test with respect to SI units** | **1.74** | **1.27** | **0.91** | **0.77** | **0.69** | **0.57** | **0.47** | **0.50** | **0.49** | **1.11** |

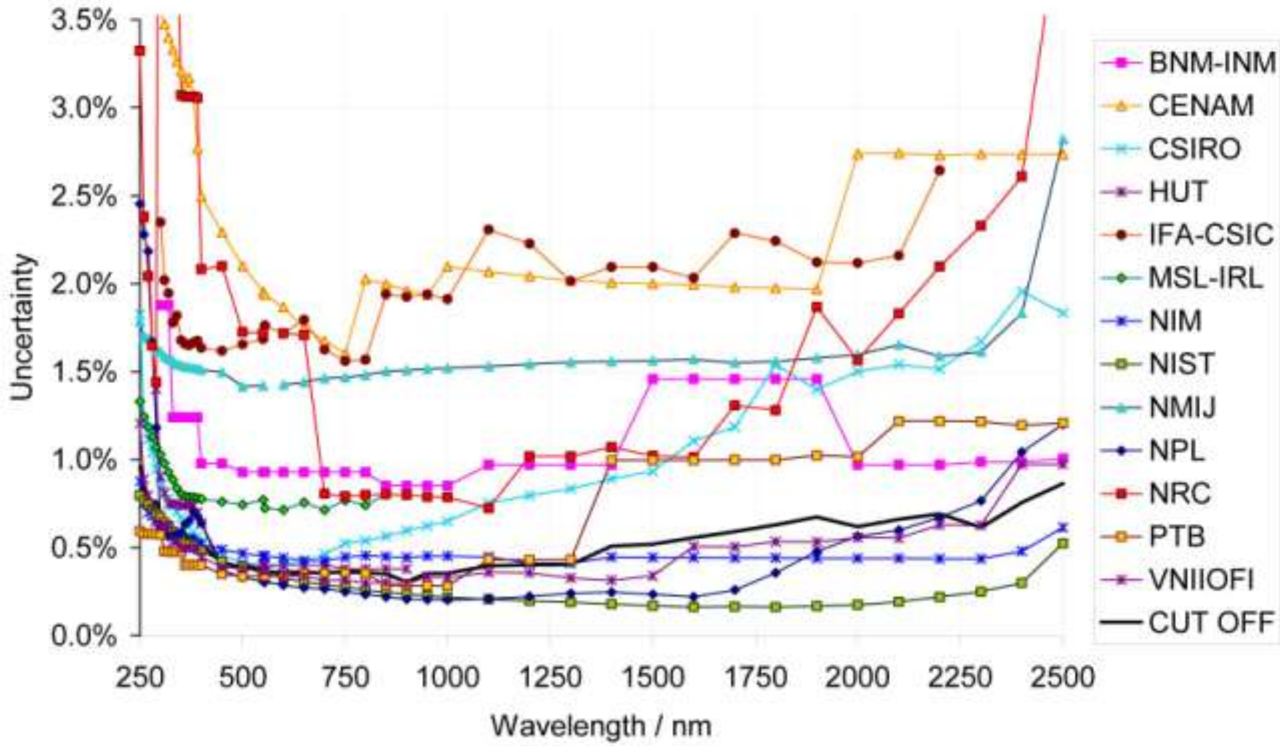
**Note:** The Type A or Type B evaluation of the uncertainty is indicated in parentheses.

## Comparison of the NIST spectral irradiance scale uncertainties

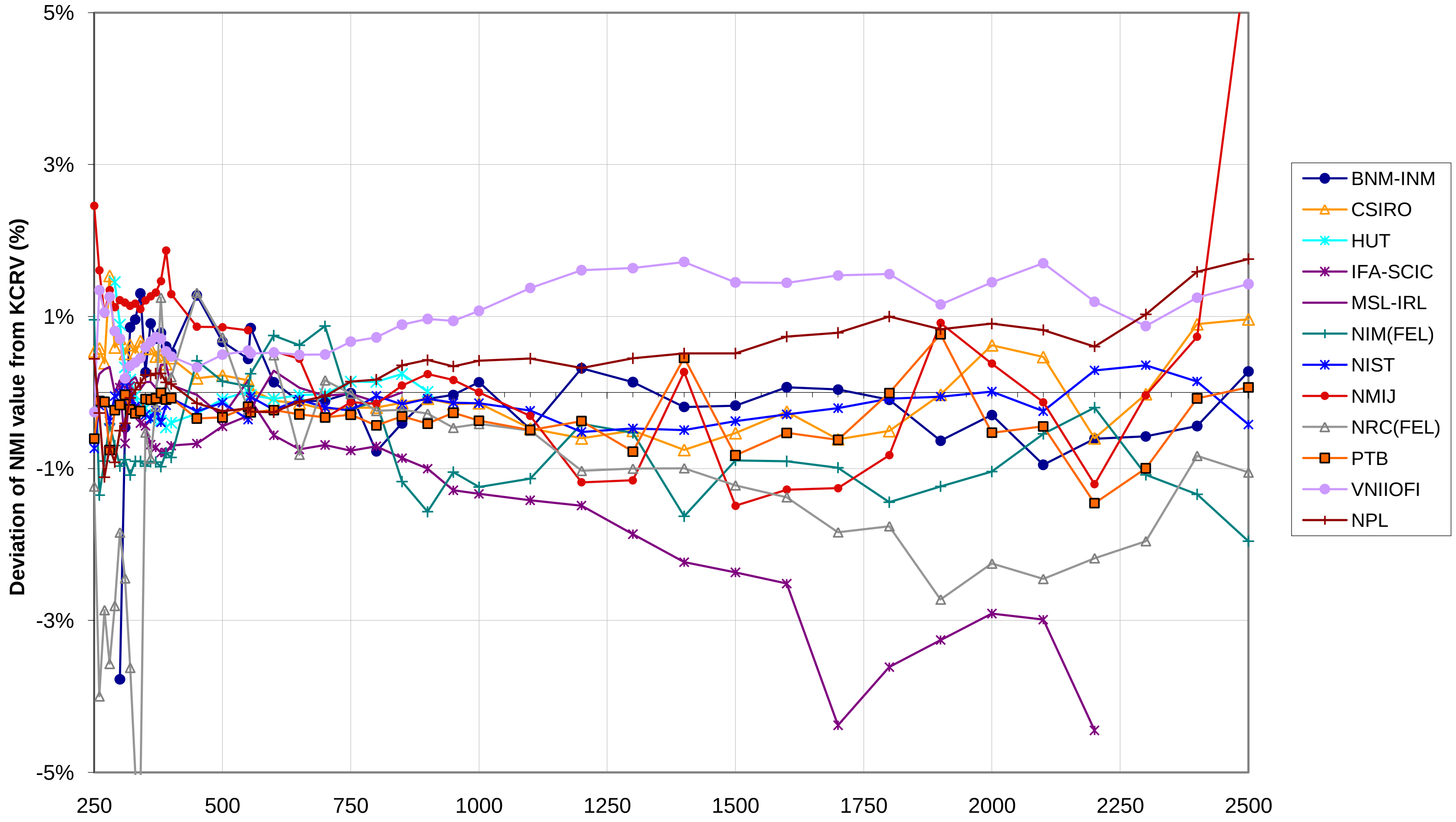



Yoon H.W., Gibson C.E., Barnes, P.Y., **The realization of the NIST detector-based spectral irradiance scale**, *Applied Optics* **41**, (2002) pp. 5879-5890.

# Uncertainties of other national labs



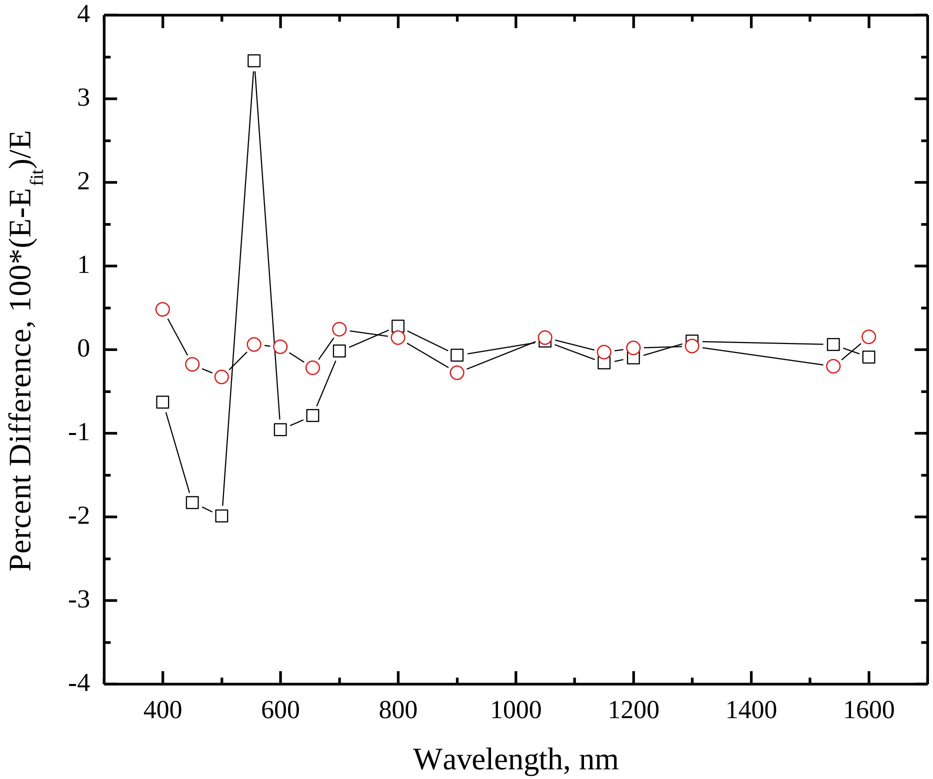
# CCPR K1a International Intercomparison Results



# Interpolations

1. Use polynomial-modified Planck fit
   A. Increase wavelength correlation
   B. Need to check the residuals
2. Use cubic spline or $4^{th}$ order Lagrange fit
   A. Will go through the points
   B. Local fit
3. Uncertainties due to the interpolations

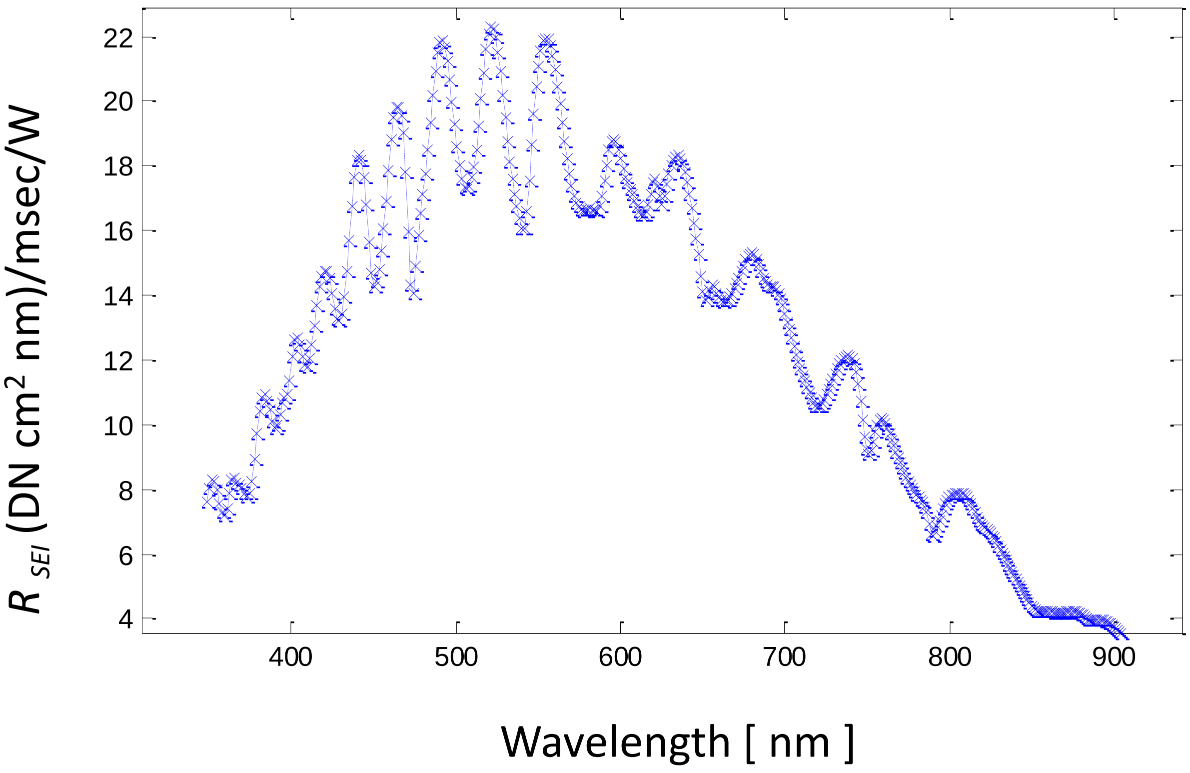
# Residuals of NIST fitting




Result of spectral irradiance value in SP250-20
At 555 nm, 107 W/cm^3
Should be 102 W/cm^3.

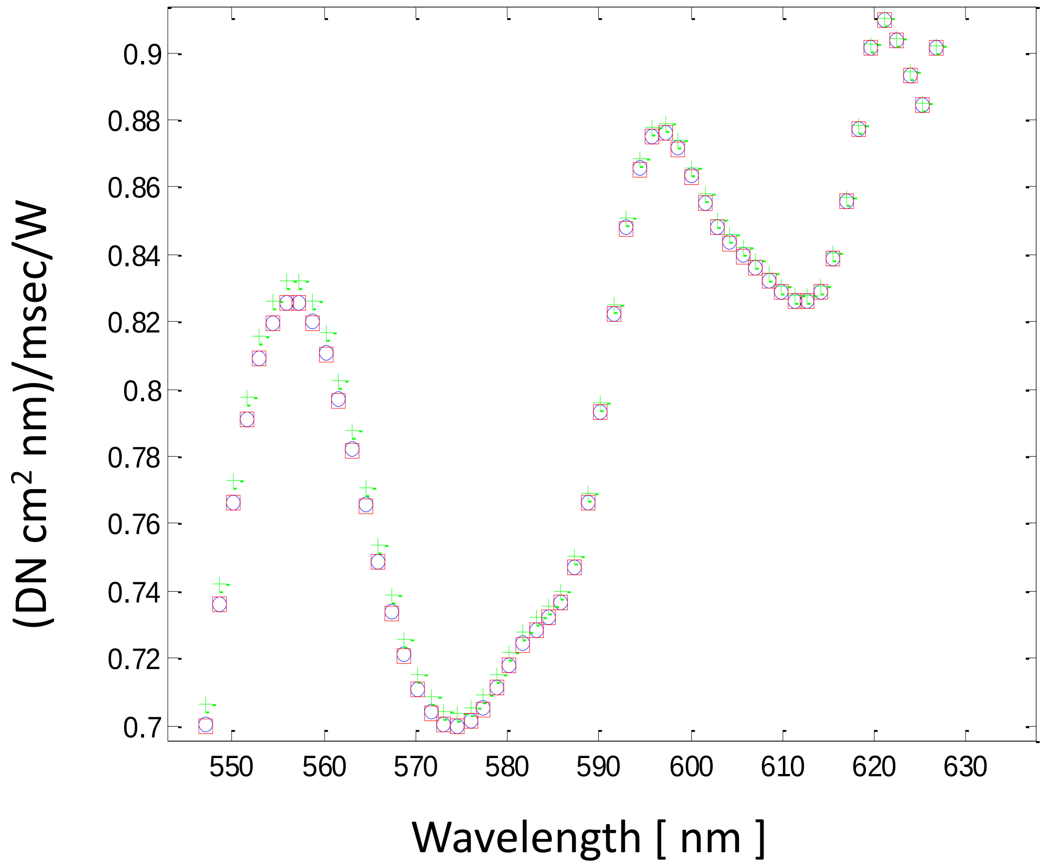
Conclusion:
Interpolate using the NIST function and cubic spline and compare the results

# Irradiance responsivity of a CCD array spectrograph




$$\frac{1}{E_U(\lambda)}\frac{\partial E_U(\lambda)}{\partial \lambda} = \frac{1}{S_U(\lambda)}\frac{\partial S_U(\lambda)}{\partial \lambda} - \frac{1}{R_{SEI}(\lambda)}\frac{\partial R_{SEI}(\lambda)}{\partial \lambda}.$$

# Wavelength repeatability not accuracy is needed




Transfer from one FEL to another FEL

# Conclusions

1. Write down a measurement equation which describes the setup.
2. Perform auxiliary measurements to determine the uncertainties.
3. Assess the accuracy of the uncertainty budget with external assessments.
4. Examine the uncertainty budget to determine if all elements are needed.