CORM 2011 NIST Measurement Uncertainty Workshop
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Case Study: Integrated goniometric measurements

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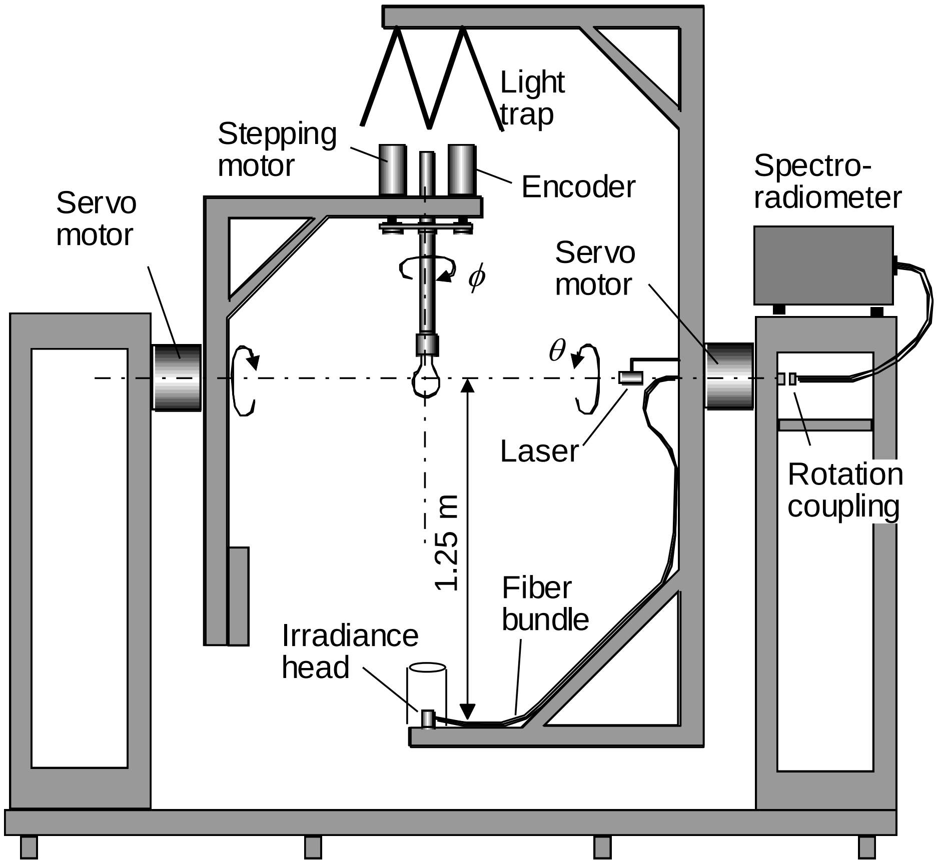
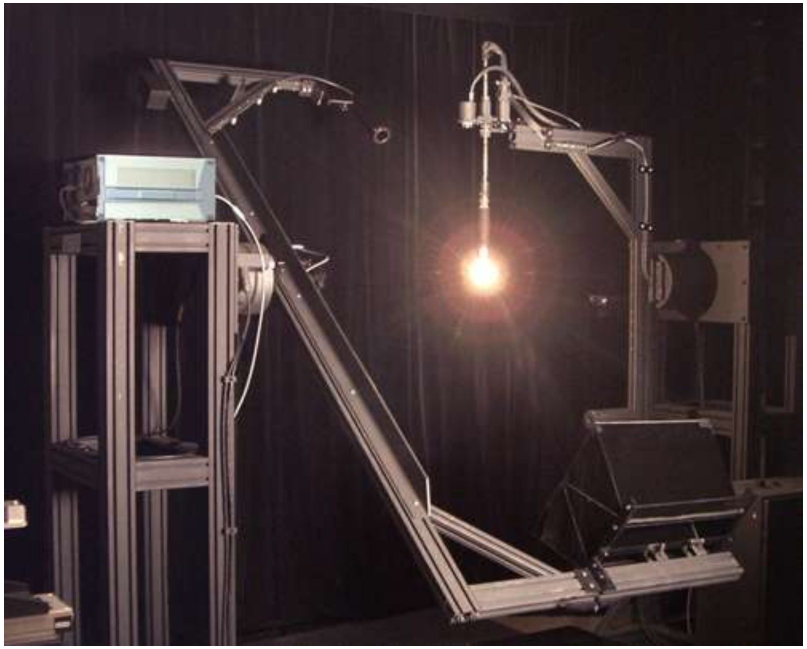
Goniometric measurements

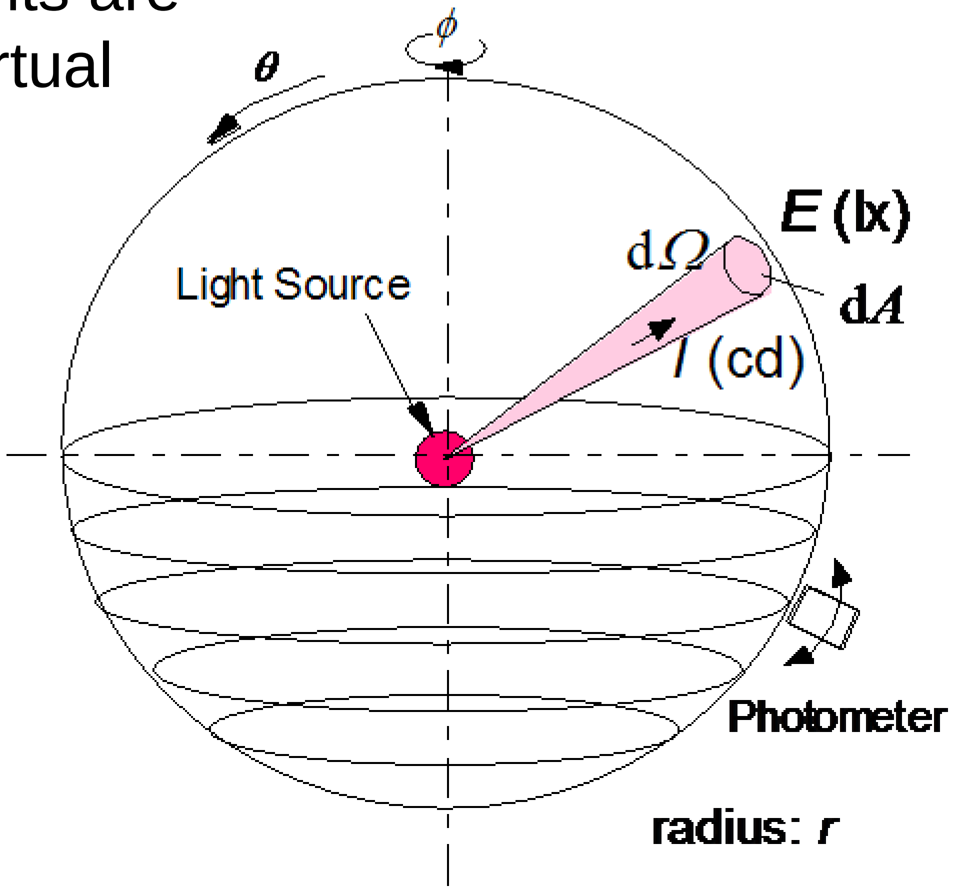


- Total luminous flux
- Total spectral radiant flux



Uncertainty calculations - flux

Goniophotometric measurements are multiple points measured on virtual sphere

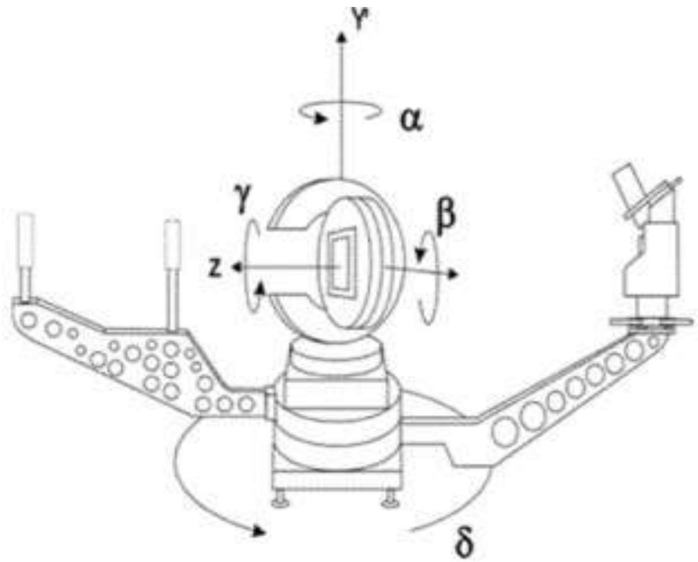


$$\Phi = \int_A E \mathrm{d}A$$

$$\Phi = r^2 \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} E(\theta,\phi) \sin\theta \, \mathrm{d}\theta \, \mathrm{d}\phi$$

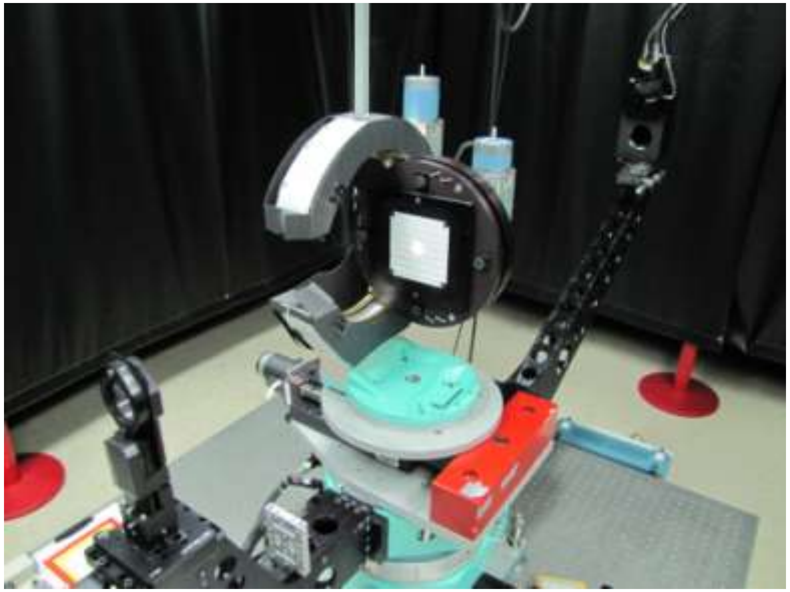
Problem: Sampling 1 % - 5 % of sphere surface

NIST Five-Axis Goniometer

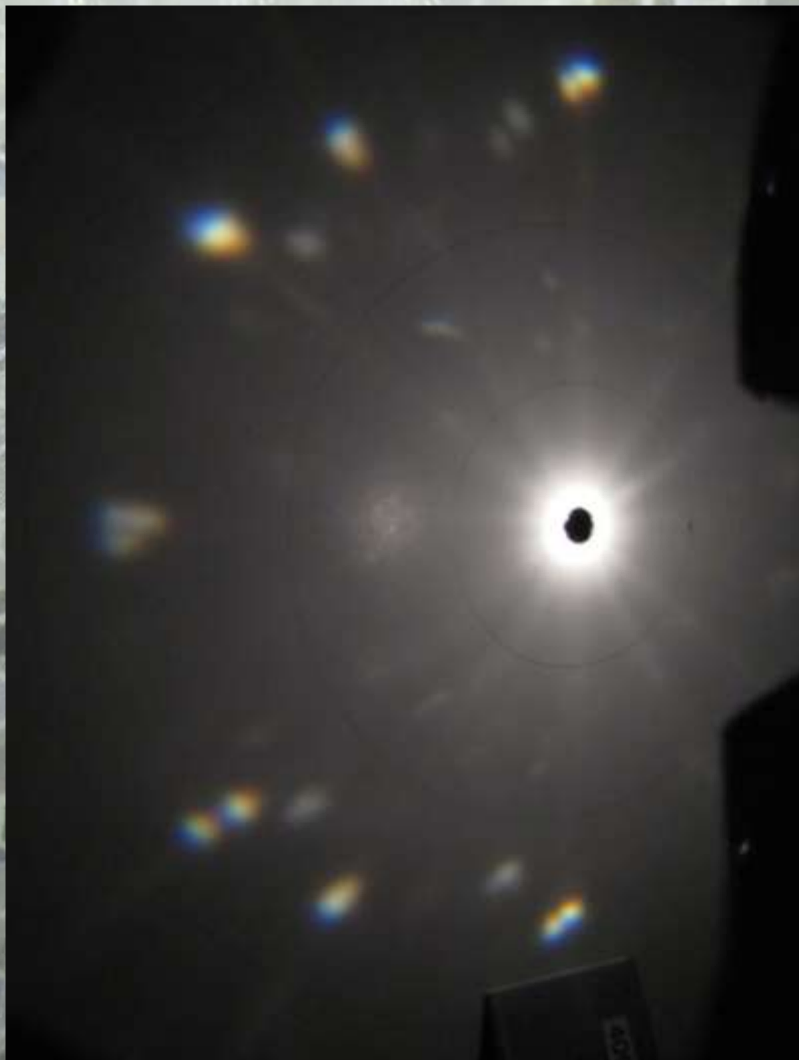


- Tool for positioning and detecting reflected light from a sample
- Can position the detector along one axis, δ
- Can position the sample along three axes, α, β, γ

- Used to take reflectance measurements over an area simulating reflectometer geometries
- Gonio measurements take a long time!

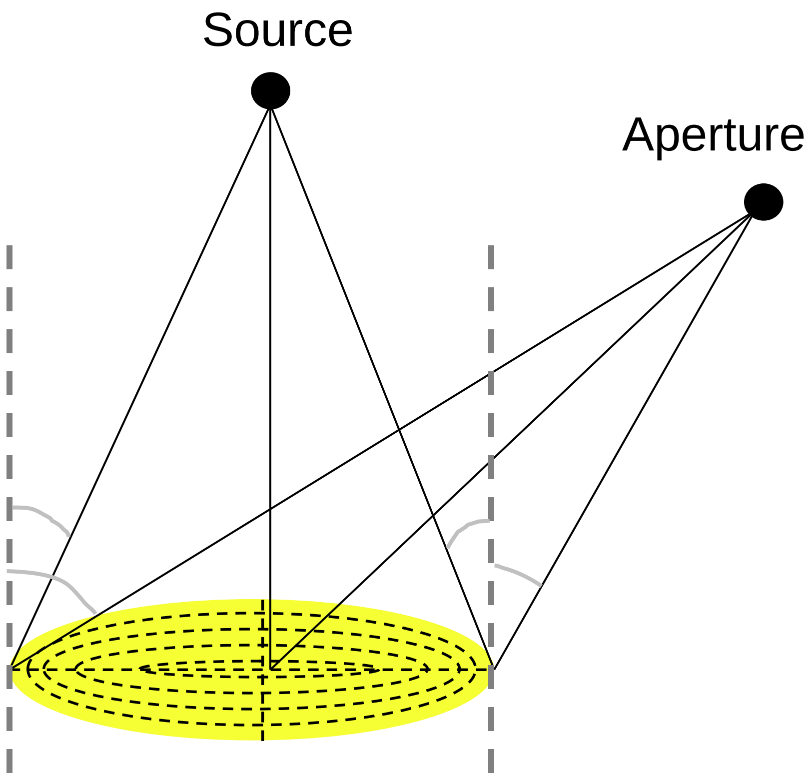


Retroreflective materials



- Micro-prismatic array
  - Highly engineered material
  - ASTM Type 4, high intensity
  - Made up of cube-corner reflectors superimposed on a hexagonal lattice grid
  - Used for medium to long distance signage and other applications

Geometry

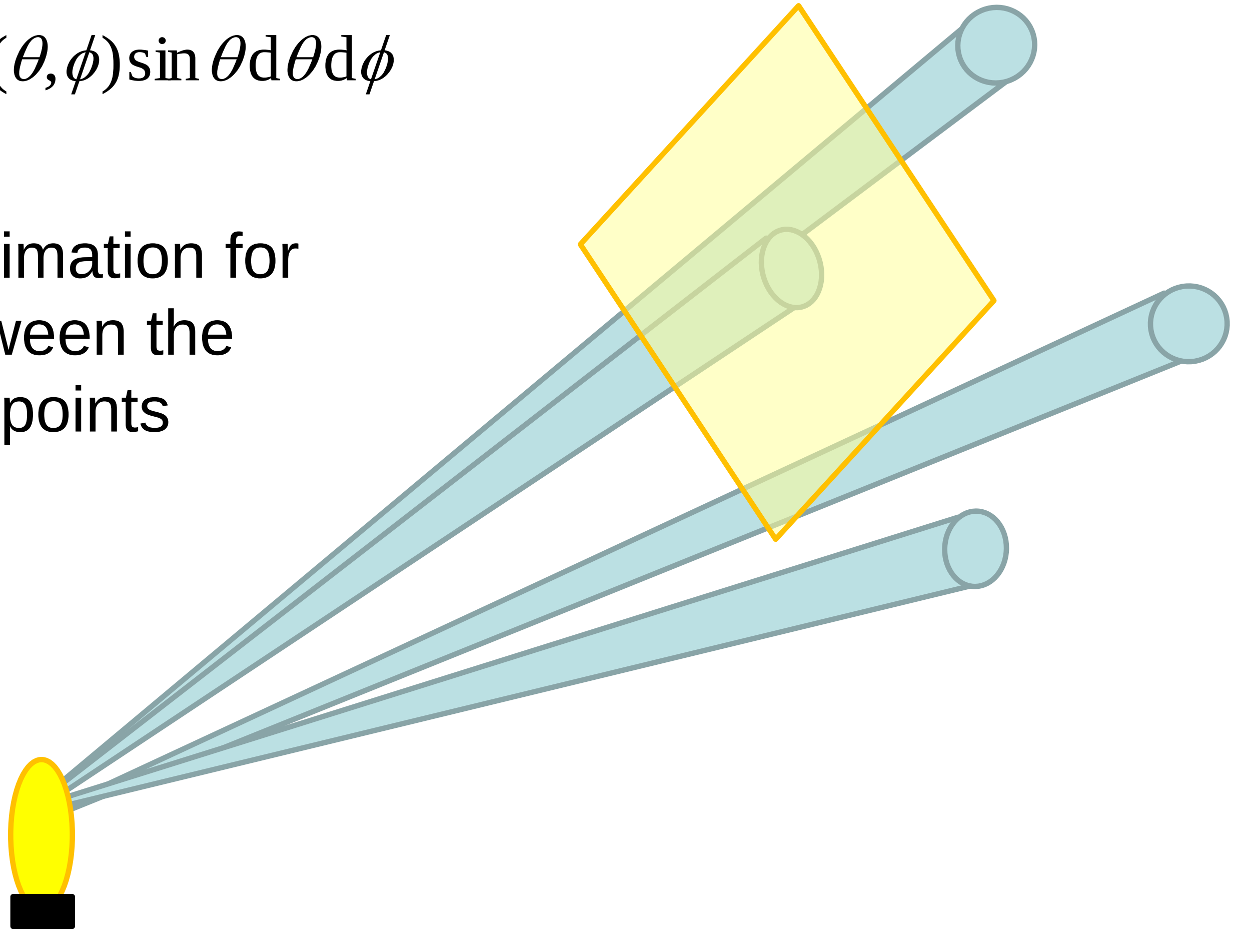


- Must make measurements at every point within the viewing aperture
- Simulates annular measurements
- Found that the average reflectance factor along one line is the same as the average reflectance within the entire area
- Scan both 0 /45 and 45 /0 geometries
- After scanning one line across the area, rotate the sample and scan another line
  - Repeat for rotation angles γ = -5 to 50
  - Sample geometry has 45 axes of symmetry

Uncertainty calculations - flux

$$\Phi = r^2 \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} E(\theta,\phi) \sin\theta \, d\theta \, d\phi$$

- Linear approximation for the area between the measured points



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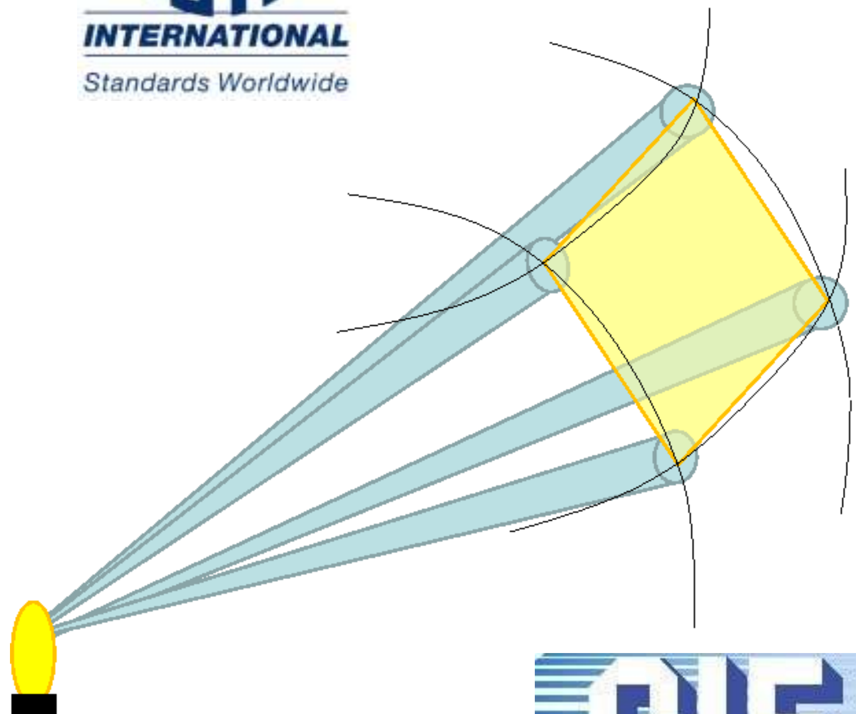
Computational optimization

- How can we reduce the data taking time?
- Since we can't measure EVERY point, we need to interpolate
- How many points do we need for a successful interpolation with minimal error?
- ASTM, CIE, and IES guidelines
  - You can never take too many data points but don't take points fewer than every 22.5 azimuthal angle (φ) and every 10 polar angle (θ).



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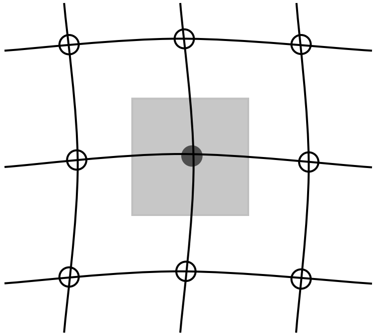
Standards Worldwide

http://www.iesna.org/

http://www.cie.co.at/

Using Mathematica



- For each point in each data set, completed a 2-D polynomial fit to the model using the Levenberg-Marquardt method $k_0 + k_1\phi + k_2\theta + k_3\phi^2 + k_4\phi\theta + k_5\theta^2$
- Prediction Bands for upper and lower bounds
- Found average value of fitted function and bounds over the subtended solid angle of interpolation
- Repeated for each point in each data set and all permutations of fit confidence level (68.3%, 95.5%) and relative uncertainty of measurement (0.05%, 0.075%, 0.1%, 0.2%, 0.5%)
- Additionally,
  - Computed simple point analysis $\sum_{\phi=0}^{360}\sum_{\theta=0}^{180} I(\theta,\phi)\,\Delta\Omega$

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http://www.wolfram.com/products/mathematica/history.html

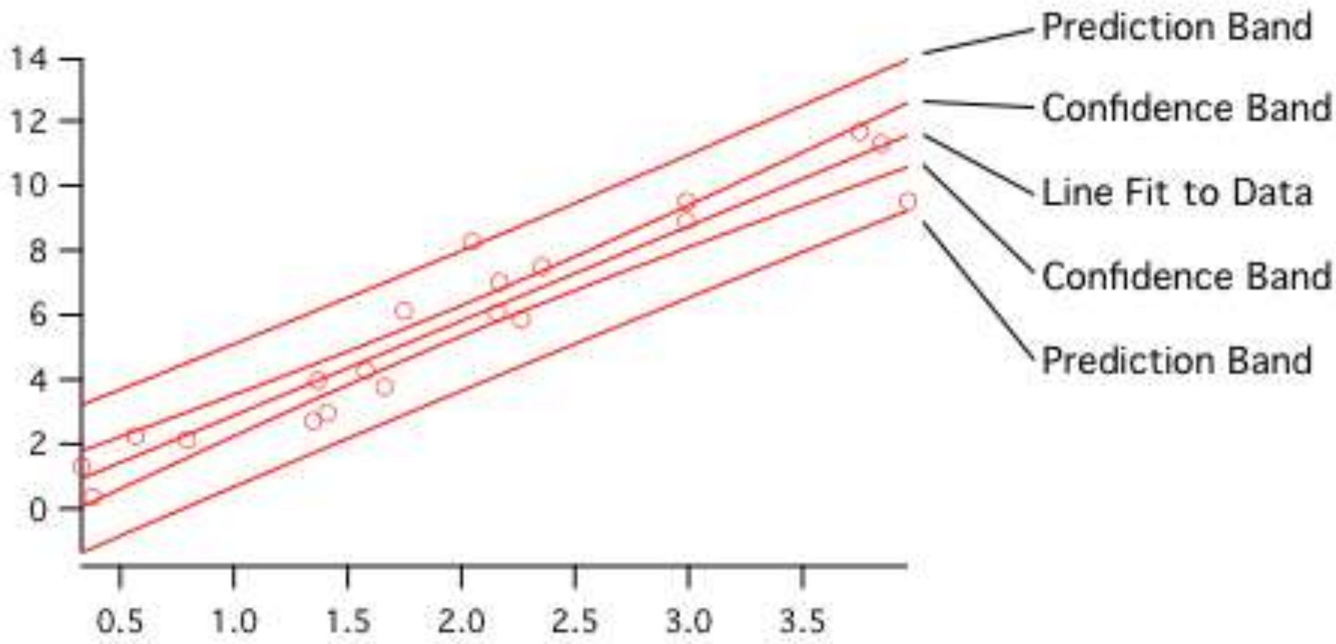
Confidence and prediction bands

Confidence and prediction bands show the region within which a model or measured data are expected to fall with a certain level of probability.

A confidence band shows the region within which the model is expected to fall.

A prediction band shows the region within which random samples from that model plus random errors are expected to fall.

Confidence and prediction bands



95% of measured points should fall within the prediction band.

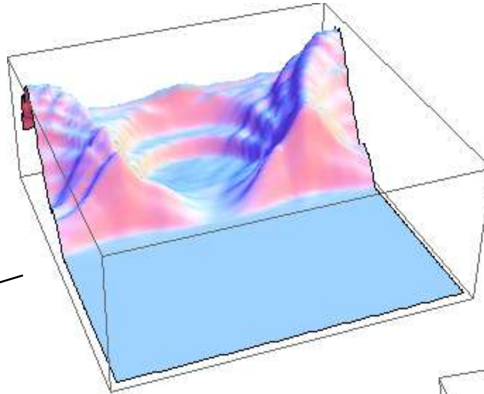
If you could repeat the experiment numerous times, 95% of the time the fit line should fall within the confidence band.

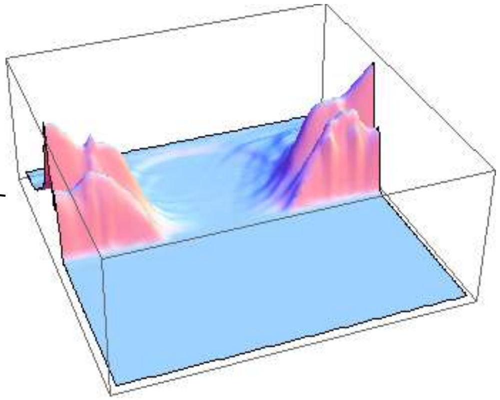
A confidence band is calculated using Student's *t* function, the coefficients of the model, and the estimated uncertainties in the coefficients. Therefore, to calculate a confidence band an estimation of uncertainty is required for each data point.

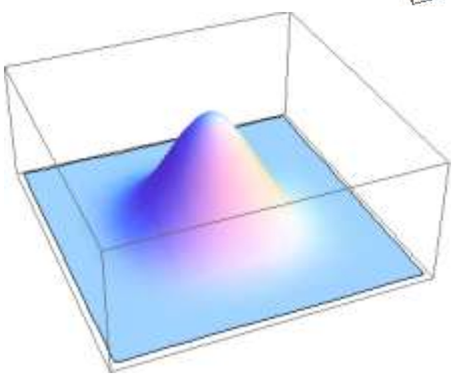
Computational Method

- Four sets of sample data at different angular spacing (θ=1,2,5,10, φ=5,22.5)
  - Real data - high intensity
    - Luminous intensity as a function of spherical angle
  - Real data - low intensity
    - Luminous intensity as a function of spherical angle
  - Fake data - constant intensity
    - Luminous intensity at a constant value of 1 candela at every spherical angle
  - Fake data - functional intensity
    - Luminous intensity according to

$$f(\theta, \phi) = \cos\left(\frac{\phi}{2} + \frac{\pi}{2}\right)^5 \cos\left(\theta + \frac{\pi}{2}\right)^5$$



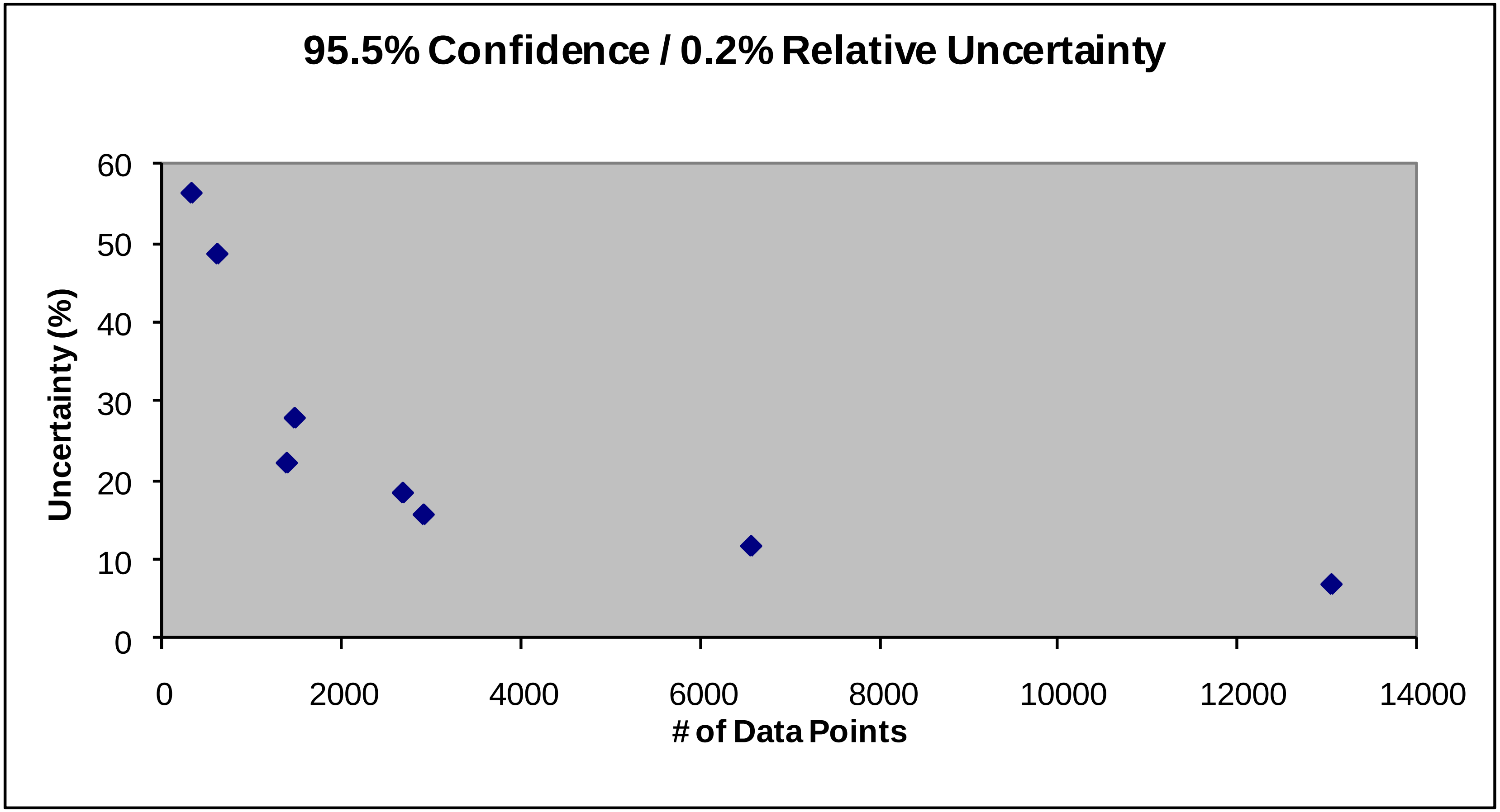




Results of Computations

- Fake data provided accurate results with very little error (<0.01% error for all)
- Real data was very structured so large uncertainties
  - Largest uncertainties are because the luminous flux was just not sampled enough to characterize the source
- Best estimates agreed reasonably well for all analyses and permutations

Results of Computations



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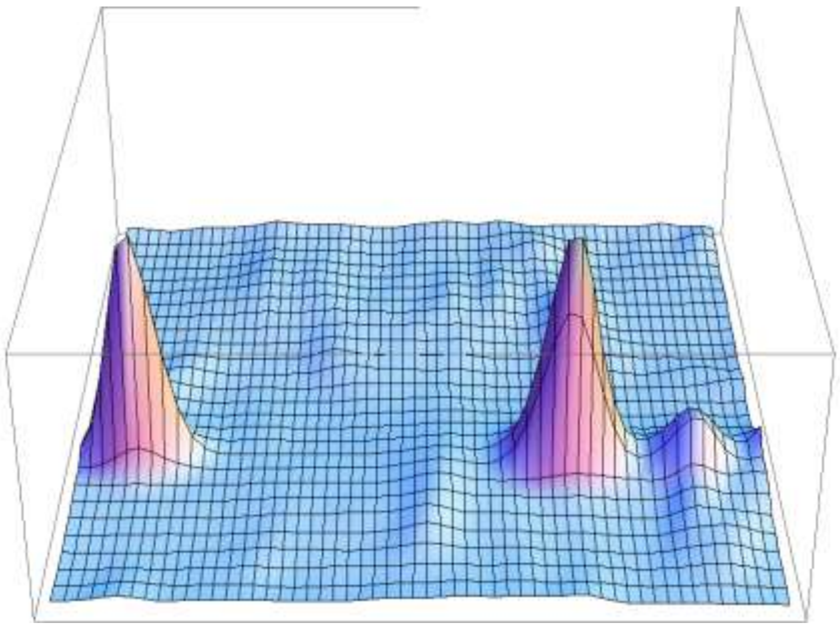
Results of Computations

% Uncertainty (68.3% Confidence, 0.1% Relative Uncertainty)				
φ \ θ	1	2	5	10
5	1.9	3.3	5.2	6.4
22.5	4.4	7.8	13.8	16.3

% Uncertainty (95.5% Confidence, 0.2% Relative Uncertainty)				
φ \ θ	1	2	5	10
5	7.0	11.8	18.6	22.5
22.5	15.8	28.1	48.8	56.5

Using the 5-Axis Goniometer

- Test scan to see the topography of the reflectance factor as a function of θ and φ
- Peaks of interest are about 5 by 10   across
- Scan every 1   in both directions to get good data
- Gonio relative uncertainty is around 0.06%
- Uncertainty from interpolation is about 1.8%
- Acceptable uncertainty
- 16 hour scan



Questions?