2017 Council of Optical Radiation Measurements
July 31st – August 2nd, 2017
Troy, NY

Uncertainty Analysis – SI-traceable Current-to-Voltage Converters

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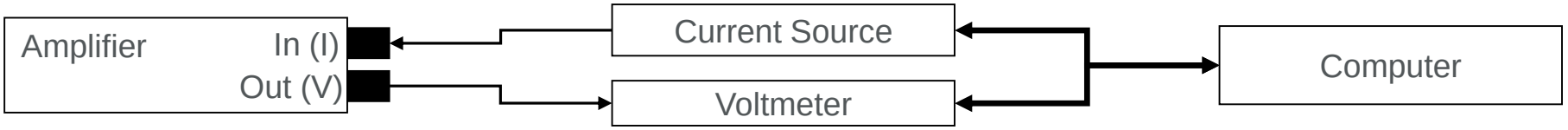
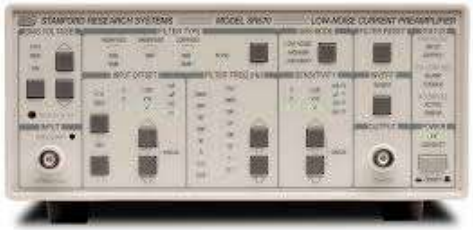
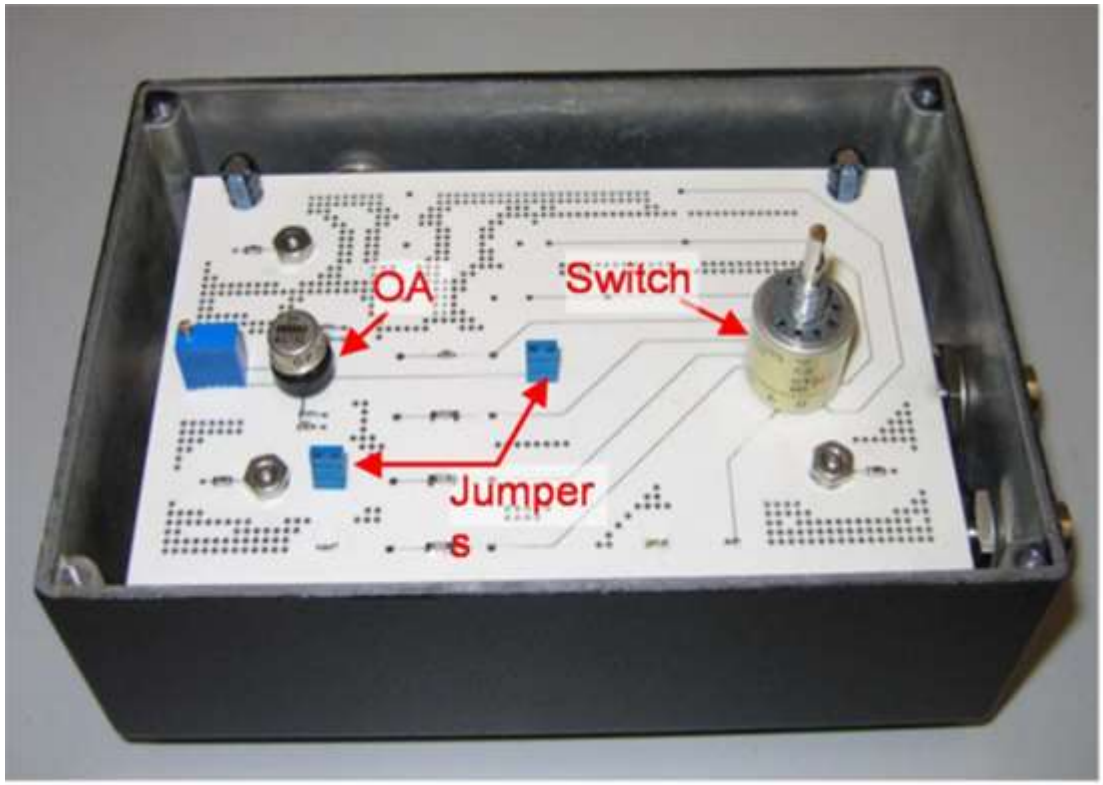
NIST

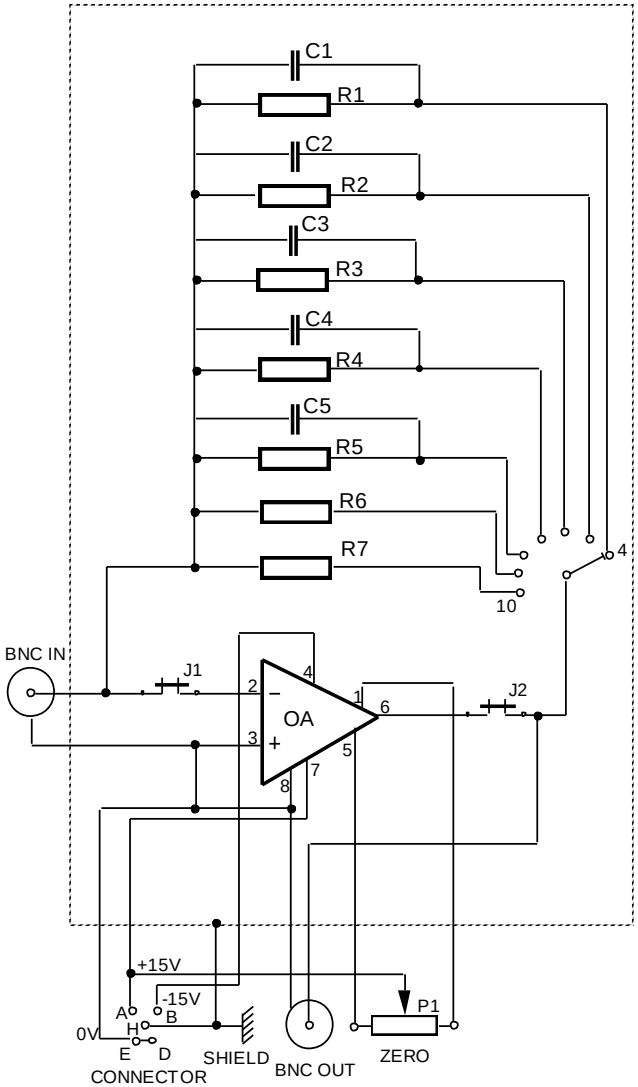
Calibration facility



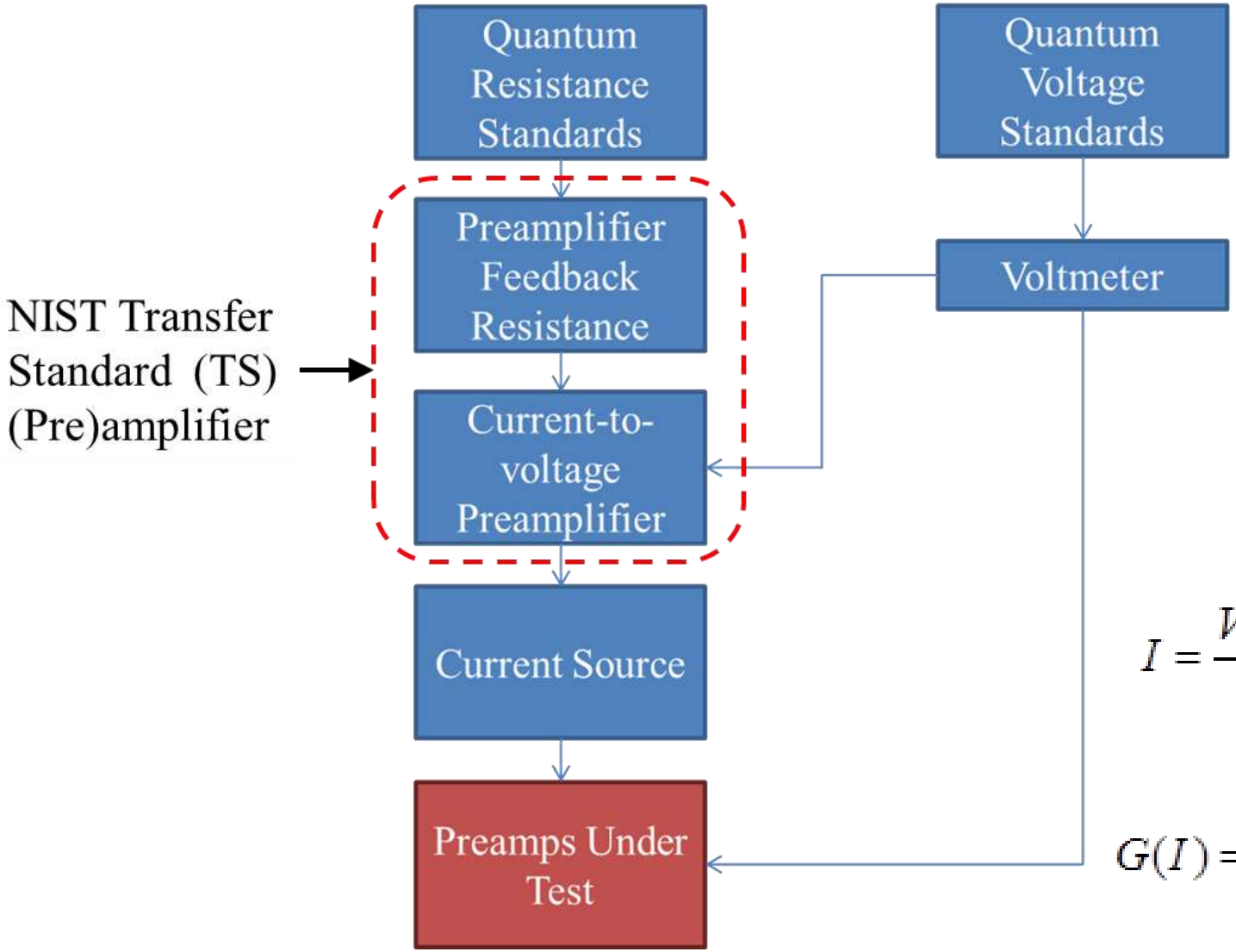


Standard current-to-voltage converter





SI-traceability of current-to-voltage converters



$$I = \frac{V_1(I) - V_1(I=0)}{R}$$

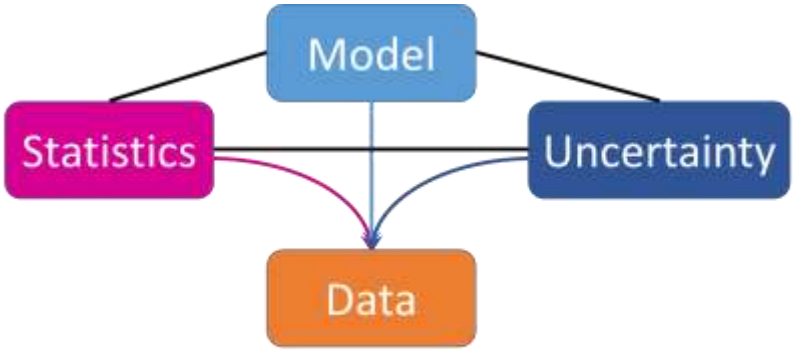
$$G(I) = \frac{V_2(I) - V_2(I=0)}{I}$$

Preamplifier feedback resistance calibration

Resistor Serial No.	Nominal Value (Ω)	Calibration Date	Calibration Temp. (°C)	Calibration Voltage	Correction (ppm)	Uncert. ppm (*k*=2)	Change (ppm)
1000-10K	1×10^4	11/18/15	22.99	10 V, 1 mA	17.4	0.3	+13.1
1010-10K	1×10^4	11/18/15	22.98	10 V, 1 mA	43.8	0.3	+12.3
1000-100K	1×10^5	11/28/15	23.00	10	57.0	0.8	+6.9
1010-100K	1×10^5	11/28/15	23.00	10	91.2	0.8	+8.4
1000-1M	1×10^6	11/12/15	23.00	10	46.2	0.8	+19.1
1010-1M	1×10^6	11/12/15	23.00	10	55.4	0.8	+21.6
1000-10M	1×10^7	11/10/15	22.99	10	147	6	+44
1010-10M	1×10^7	11/10/15	22.99	10	100	6	+22
1000-100M	1×10^8	11/13/15	22.98	10	57	6	+45
1010-100M	1×10^8	11/13/15	22.98	10	113	6	+36
1000-1G	1×10^9	11/8/15	23.00	10	841	18	+12
1010-1G	1×10^9	11/8/15	23.00	10	-282	18	+11
1000-10G	1×10^{10}	11/4/15	23.01	10	857	50	-40
1010-10G	1×10^{10}	11/4/15	23.01	10	-18	50	-30

Voltmeter calibration

The DVM's internal reference was measured as 7.08346213 V before the adjustment. The internal voltage reference is calibrated by a solid-state voltage standard which in turn was calibrated using the 10 V Josephson Voltage Standard. The known voltage from Fluke 732B SN 9750710 was used to calibrate the DVM internal reference. A new internal reference voltage value was assigned as 7.08346152 V after the adjustment.



Weighted fitting & Chi Square Test (goodness of fit)

DVM Reading (V)	Array Voltage (V)	DVM Error (µV)	Linearity Dev. (µV/V)
-10.002 610 014	-10.002 609 330	-.680	-.970
-9.003 154 143	-9.003 154 282	.140	-.140
-8.004 330 770	-8.004 331 300	.530	.260
-7.003 611 229	-7.003 612 119	.890	.630
-6.002 418 394	-6.002 418 888	.490	.240
-5.004 385 612	-5.004 385 989	.380	.130
-4.003 192 297	-4.003 192 758	.460	.220
-3.003 579 344	-3.003 579 693	.350	.110
-2.004 914 258	-2.004 914 728	.470	.240
-1.004 290 166	-1.004 290 357	.190	-.030
-.001 706 546	-.001 706 579	.030	-.180
.996 295 102	.996 294 716	.390	.180
1.999 953 028	1.999 953 006	.020	-.180
2.998 207 193	2.998 207 128	.060	-.130
3.997 820 354	3.997 820 193	.160	-.030
4.999 487 217	4.999 487 474	-.260	-.440
5.996 414 150	5.996 414 257	-.110	-.280
6.998 397 728	6.998 397 571	.160	-.010
7.998 958 869	7.998 958 735	.130	-.030
8.999 204 235	8.999 203 866	.370	.210
9.998 659 288	9.998 658 915	.370	.220

Fitting rigor – Chi square goodness of fit test

Null hypothesis – the model represents the distribution described by the measured data points

Chi-square test as a 'goodness of fit'

Based on a confidence level

($\alpha < 0.05$)

Degree of freedom

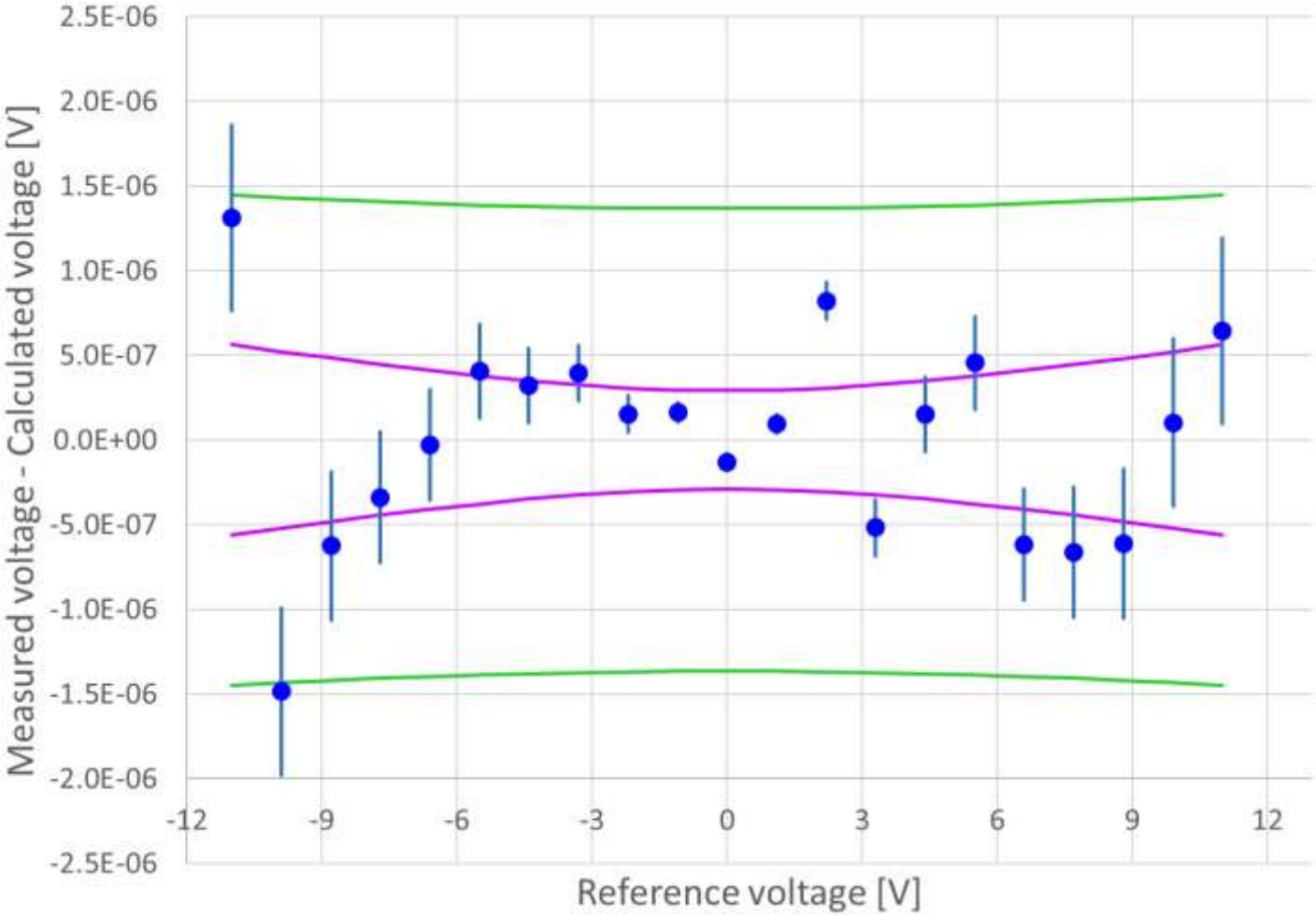
($\nu = 19$,

21 points – 2 parameters)

→ Critical value 30.144

	Probability less than the critical value (1- α)				
ν	0.90	0.95	0.975	0.99	0.999
1	2.706	3.841	5.024	6.635	10.828
2	4.605	5.991	7.378	9.210	13.816
3	6.251	7.815	9.348	11.345	16.266
4	7.779	9.488	11.143	13.277	18.467
5	9.236	11.070	12.833	15.086	20.515
6	10.645	12.592	14.449	16.812	22.458
7	12.017	14.067	16.013	18.475	24.322
8	13.362	15.507	17.535	20.090	26.125
9	14.684	16.919	19.023	21.666	27.877
10	15.987	18.307	20.483	23.209	29.588
11	17.275	19.675	21.920	24.725	31.264
12	18.549	21.026	23.337	26.217	32.910
13	19.812	22.362	24.736	27.688	34.528
14	21.064	23.685	26.119	29.141	36.123
15	22.307	24.996	27.488	30.578	37.697
16	23.542	26.296	28.845	32.000	39.252
17	24.769	27.587	30.191	33.409	40.790
18	25.989	28.869	31.526	34.805	42.312
19	27.204	30.144	32.852	36.191	43.820
20	28.412	31.410	34.170	37.566	45.315

Voltage gain calibration



Uncertainty
0.025 ppm

Chi-square
26.67

Gain
0.999999872(21)

Offset
0.00000059(14)

Confidence & prediction bands

$$CI(x^*) = t_{\alpha,n-2} \cdot \sqrt{\boldsymbol{a}^T \boldsymbol{C} \boldsymbol{a}}$$

$$PI(x^*) = t_{\alpha,n-2} \cdot \sqrt{\sum \frac{(y_i - \hat{y}_i)^2}{n-2} + \boldsymbol{a}^T \boldsymbol{C} \boldsymbol{a}}$$

$t_{\alpha,n-2}$ - the Student's t-distribution

α - probability or confidence level

n - degrees of freedom

$\boldsymbol{C}$ - covariance matrix

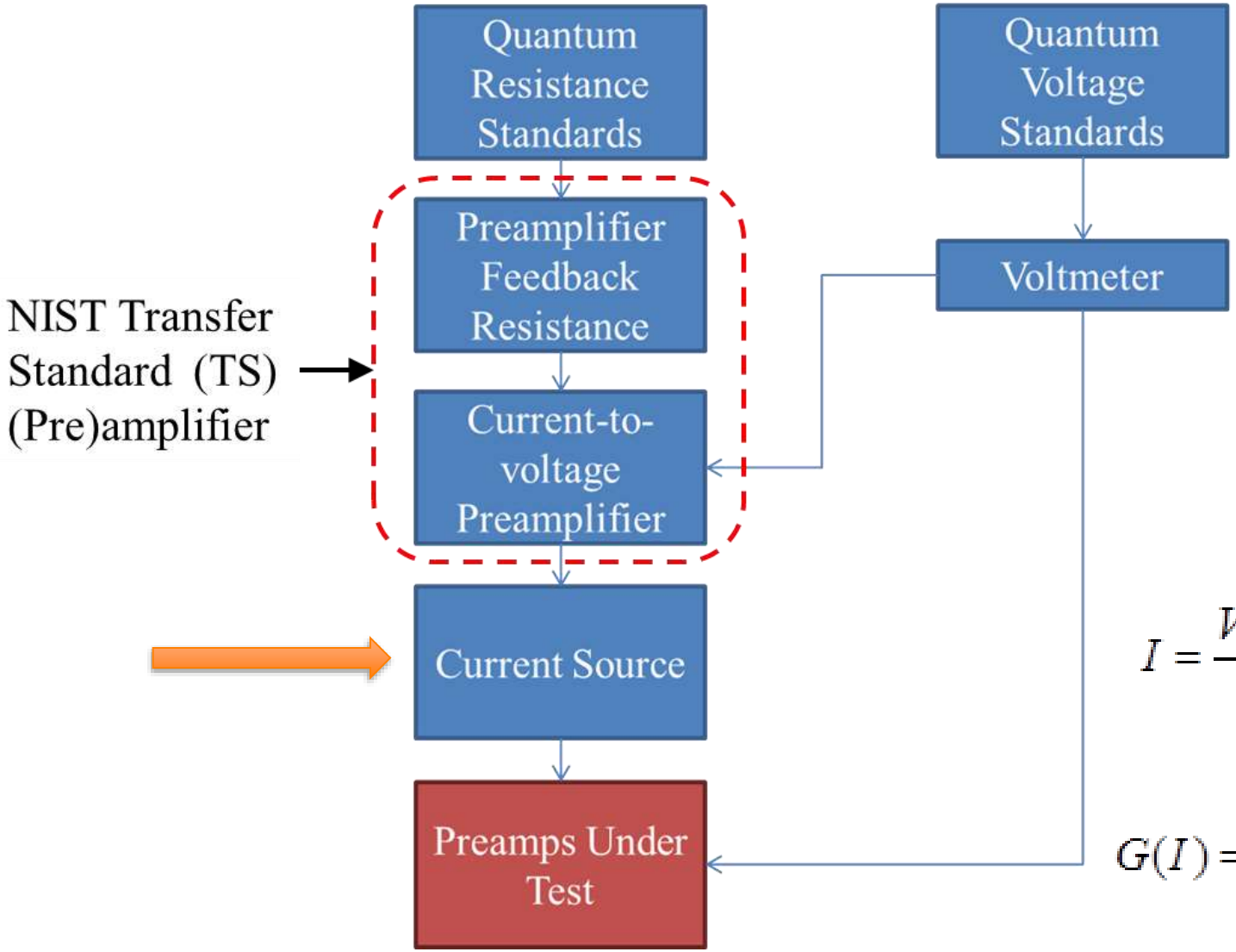
$\boldsymbol{a}$ - vector of the sensitivity coefficients

$$\boldsymbol{a}^T \boldsymbol{C} \boldsymbol{a} = \begin{bmatrix} \frac{\partial y}{\partial m} & \frac{\partial y}{\partial b} \end{bmatrix} \begin{bmatrix} \sigma_m^2 & Cov(m,b) \\ Cov(m,b) & \sigma_b^2 \end{bmatrix} \begin{bmatrix} \frac{\partial y}{\partial m} \\ \frac{\partial y}{\partial b} \end{bmatrix}$$

$$\boldsymbol{a}^T \boldsymbol{C} \boldsymbol{a} = x^{*2} \sigma_m^2 + \sigma_b^2 + 2x^* Cov(m,b)$$

$$Cov(m,b) = \frac{b \cdot \sigma_m \cdot \sigma_b \sqrt{\sum_j \left(\frac{1}{\sigma_j^2} \cdot \left(\sum_i \frac{x_i^2}{\sigma_i^2} \right) \right) - \left(\sum_i \frac{x_i}{\sigma_i^2} \right)^2}}{\sum_j \left(\frac{1}{\sigma_j^2} \cdot \left(\sum_i \frac{y_i^2}{\sigma_i^2} \right) \right) - \left(\sum_i \frac{y_i}{\sigma_i^2} \right)^2}$$

SI-traceability of current-to-voltage converters



$$I = \frac{V_1(I) - V_1(I=0)}{R}$$

$$G(I) = \frac{V_2(I) - V_2(I=0)}{I}$$

Current source calibration

		Absolute uncertainty [V x 10^{-6}]						
Uncertainty components	**Type**	**Range $1x10^4$ V/A**	**$1x10^5$ V/A**	**$1x10^6$ V/A**	**$1x10^7$ V/A**	**$1x10^8$ V/A**	**$1x10^9$ V/A**	**$1x10^{10}$ V/A**
Feedback resistance	B	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Measurement noise	A	6.17	19.6	24.7	18.3	46.2	36.3	227
Voltage measurement	B	0.14	0.14	0.14	0.14	0.14	0.14	0.14
Repeatability	A	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Long-term drift of feedback resistance	A	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Loop gain	B	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Combined absolute standard uncertainty term of *I* measurement		6.17	19.6	24.7	18.3	46.2	36.3	227

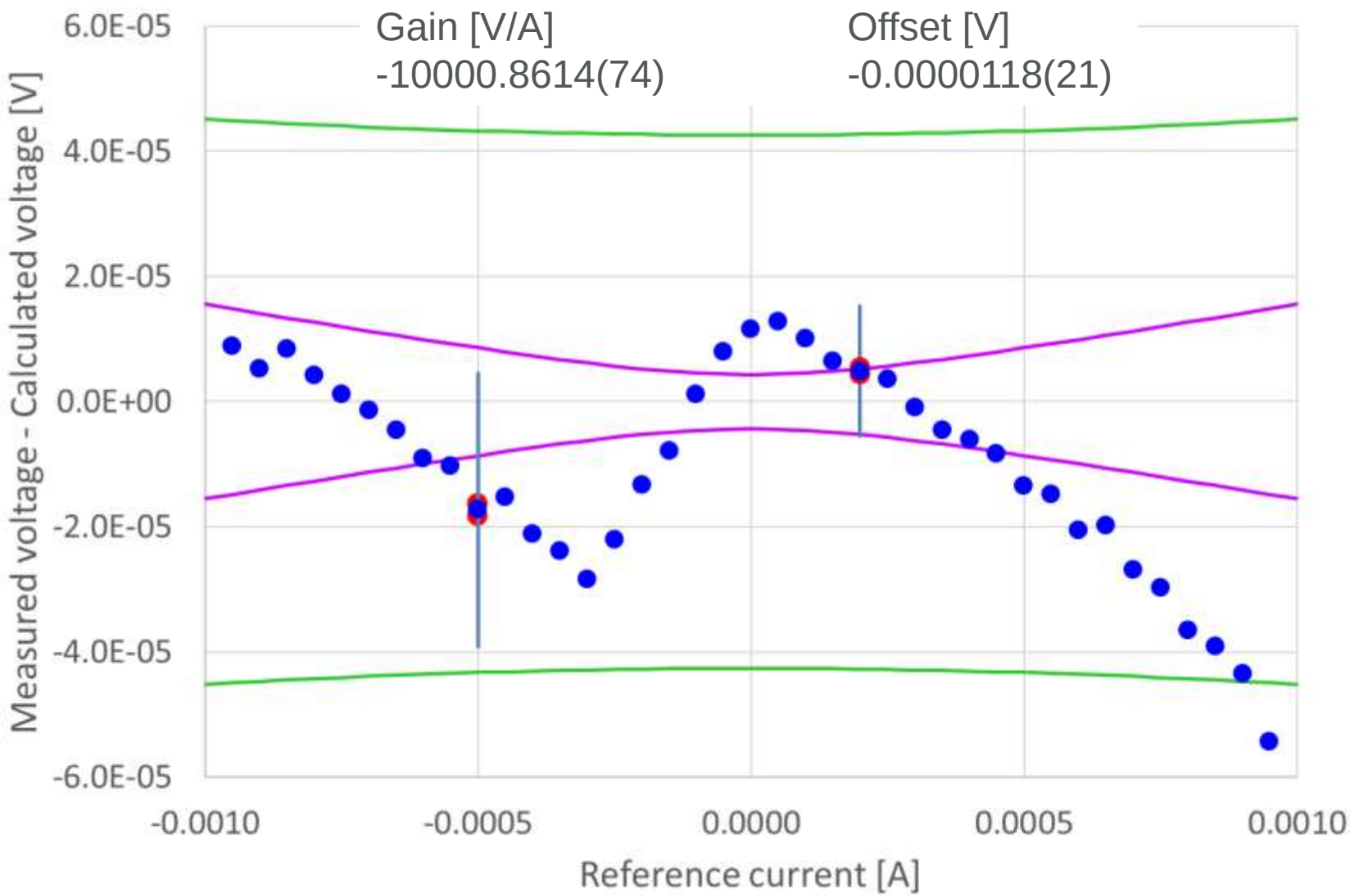
		Relative uncertainty [ppm]						
Uncertainty components	**Type**	**Range $1x10^4$ V/A**	**$1x10^5$ V/A**	**$1x10^6$ V/A**	**$1x10^7$ V/A**	**$1x10^8$ V/A**	**$1x10^9$ V/A**	**$1x10^{10}$ V/A**
Feedback resistance	B	0.15	0.40	0.40	3.00	3.00	9.00	25.0
Measurement noise	A	0.16	0.85	0.93	0.94	1.37	7.37	55.6
Voltage measurement	B	0.01	0.01	0.01	0.01	0.01	0.01	0.01
Repeatability	A	1.16	3.84	4.08	4.63	3.58	10.0	61.2
Long-term drift of feedback resistance	A	2.13	1.45	3.74	3.81	6.23	1.90	5.19
Loop gain	B	2.00	2.00	2.00	2.00	2.00	2.00	2.00
Combined relative standard uncertainty term of *I* measurement		3.15	4.66	5.97	7.06	8.16	15.6	86.6

Device under test uncertainty budget

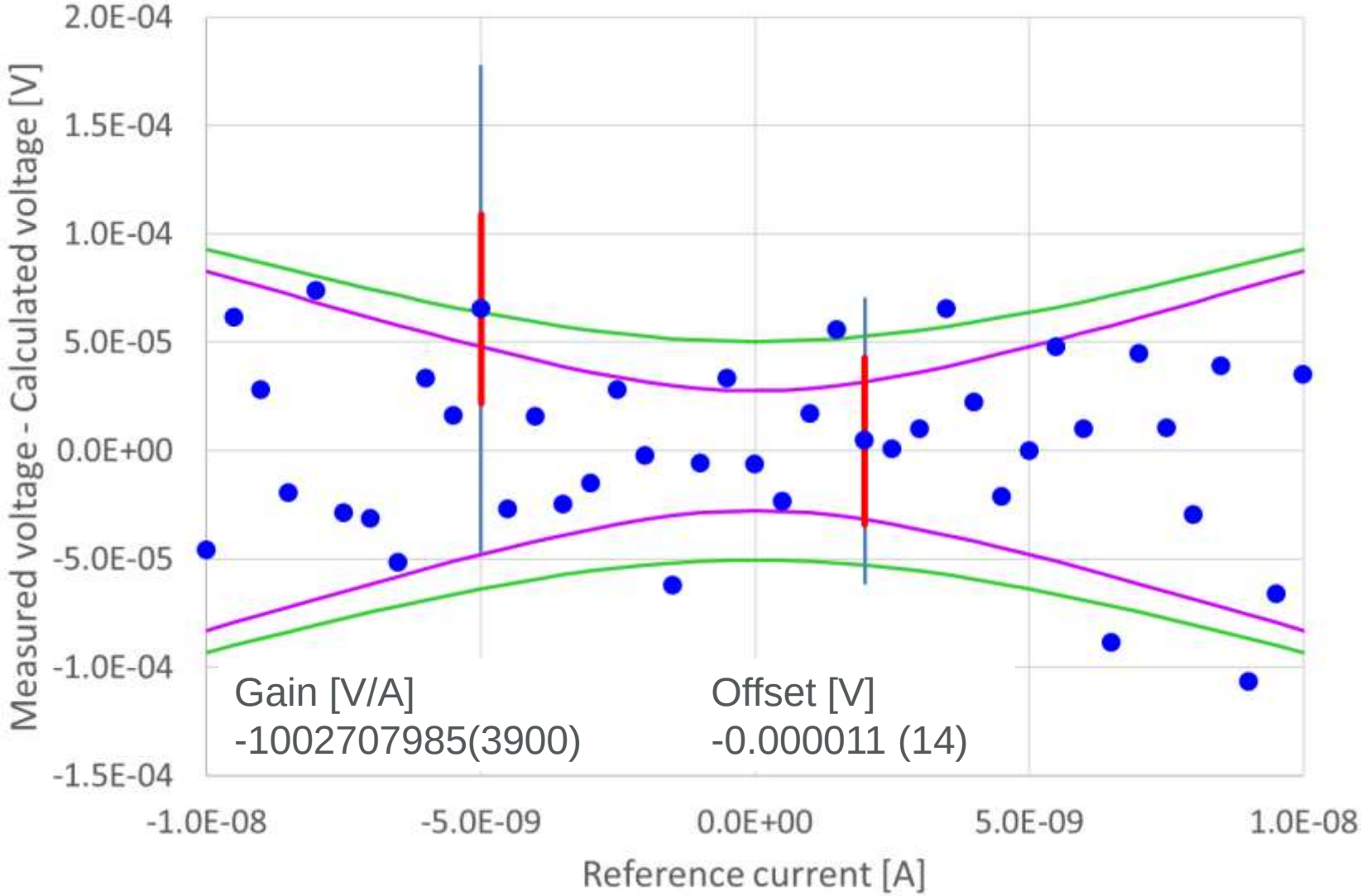
		Absolute uncertainty [V x 10^{-6}]					
Uncertainty components	**Type**	**Range $1x10^4$ V/A**	**$1x10^5$ V/A**	**$1x10^6$ V/A**	**$1x10^7$ V/A**	**$1x10^8$ V/A**	**$1x10^9$ V/A**
Current	B	6.17	19.6	24.7	18.3	46.2	36.3
Measurement noise	A	12.5	21.0	37.0	26.4	40.6	35.0
Voltage measurement	B	0.14	0.14	0.14	0.14	0.14	0.14
Reproducibility (2 months)	A	0.00	0.00	0.00	0.00	0.00	0.00
Loop gain	B	0.00	0.00	0.00	0.00	0.00	0.00
Combined absolute standard uncertainty term of *G* measurement		13.9	28.7	44.5	32.1	61.5	50.4

		Relative uncertainty [ppm]					
Uncertainty components	**Type**	**Range $1x10^4$ V/A**	**$1x10^5$ V/A**	**$1x10^6$ V/A**	**$1x10^7$ V/A**	**$1x10^8$ V/A**	**$1x10^9$ V/A**
Current	B	3.15	4.66	5.97	7.06	8.16	15.6
Measurement noise	A	0.19	0.96	0.95	0.88	1.41	5.47
Voltage measurement	B	0.01	0.01	0.01	0.01	0.01	0.01
Reproducibility (2 months)	A	9.61	10.9	6.37	5.20	8.11	12.8
Loop gain	B	2.00	2.00	2.00	2.00	2.00	2.00
Combined relative standard uncertainty term of *G* measurement		10.3	12.1	9.01	9.04	11.8	21.0

Device under test data points – Gain 4



Device under test data points – Gain 9



Determination of current

$$y^* = 5.000419\ V$$

On a gain setting of 4, what is the current measured and the uncertainty of the current?

$$x^* = \frac{(y^* - Offset)}{Gain}$$

	Quantity	Value x_i	Unit	Standard uncertainty $u(x_i)$	Type	DOF ν	Sensitivity c_i	Contribution $u_i(y)$
1	y^*	5.000419	V	???	???	???		
2	$Gain$	-10000.8614	V/A	0.0074	B	∞		
3	$Offset$	-0.0000118	V	-0.0000021	B	∞		
	x^*	5.000000	A				$u_c(x^*)$	

Certificate – uncertainty equation

$$PI(x^*) = t_{\alpha,n-2} \cdot \sqrt{\sum \frac{(y_i - \hat{y}_i)^2}{n-2} + \boldsymbol{a}^T \boldsymbol{C} \boldsymbol{a}}$$

And the uncertainty of the NIST current scale used in the calibration of the device under test

OR

$$U(y) = 2 * \sqrt{\left(\propto * \left[(y - b)/m\right]^2 + \beta\right) + (\gamma * y)^2}$$

	Linear Least Squares Fit (y = mx +b)					
Range Setting	**1x10^4 V/A**	**1x10^5 V/A**	**1x10^6 V/A**	**1x10^7 V/A**	**1x10^8 V/A**	**1x10^9 V/A**
α [A^2]	5.56E-05	8.84E-03	7.26E+00	5.78E+02	1.09E+05	1.53E+07
β [V^2]	4.54E-10	4.62E-10	6.68E-10	7.94E-10	7.98E-10	2.07E-09
γ	9.61E-06	1.09E-05	6.37E-06	5.20E-06	8.11E-06	8.64E-05

Determination of current

$$y^* = 5.000419\ V$$

On a gain setting of 4, what is the current measured and the uncertainty of the current?

$$x^* = \frac{(y^* - Offset)}{Gain}$$

	Quantity	Value x_i	Unit	Standard uncertainty $u(x_i)$	Type	DOF ν	Sensitivity c_i	Contribution $u_i(y)$
1	y^*	5.000419	V	0.0000527	B	∞	-1.00E-04	-5.27E-09
2	$Gain$	-10000.8614	V/A	0.0074	B	∞	5.00E-08	3.68E-10
3	$Offset$	-0.0000118	V	-0.0000021	B	∞	1.00E-04	2.13E-10
	x^*	0.500000E-04	A				$u_c(x^*)$	5.3E-09

$U(x^*)$ = 1.1E-08 A 21 ppm

Questions

NIST Transfer Standard I-V Converter (i.e., amplifier)