Basics of Uncertainty Estimation

Why, What,
Who, When, Where,
and How

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Why

1. To provide users of data with a quantification of expected variation

2. Regulation requirements

3. To understand the measurement process

Why

1) To provide users of data a quantification of exactness ?

- Is it ever used?
- *U* < needs (decision limits) for most non-radiometric measurements
- *U* < DUT variations for many radiometric measurements

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Why

2) Regulation requirements

- No requirements or limits, that I have seen

- This is a method of proving that you understand what you are doing,
  so it is actually reason number three

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Why

3) To understand and control the measurement process

- Discovery of errors and limits to "accuracy"

- Identification of areas to improve (low hanging fruit)

- Reduce effort on non-critical parts of the process

<u>This is the best and maybe only reason</u>

What

Quantification of our lack of knowledge

- Uncertainty is what is left over after corrections and calibrations
- Has zero mean or it would be a correction
- Meaningful units, the same as results
- Probabilistic in nature, based on confidence

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Who, When and Where

To get the benefits

Who

- Those who design and implement the measurement have the information and need the understanding

When

- During the design and measurement

Where

- Performed manually not in an automatic program

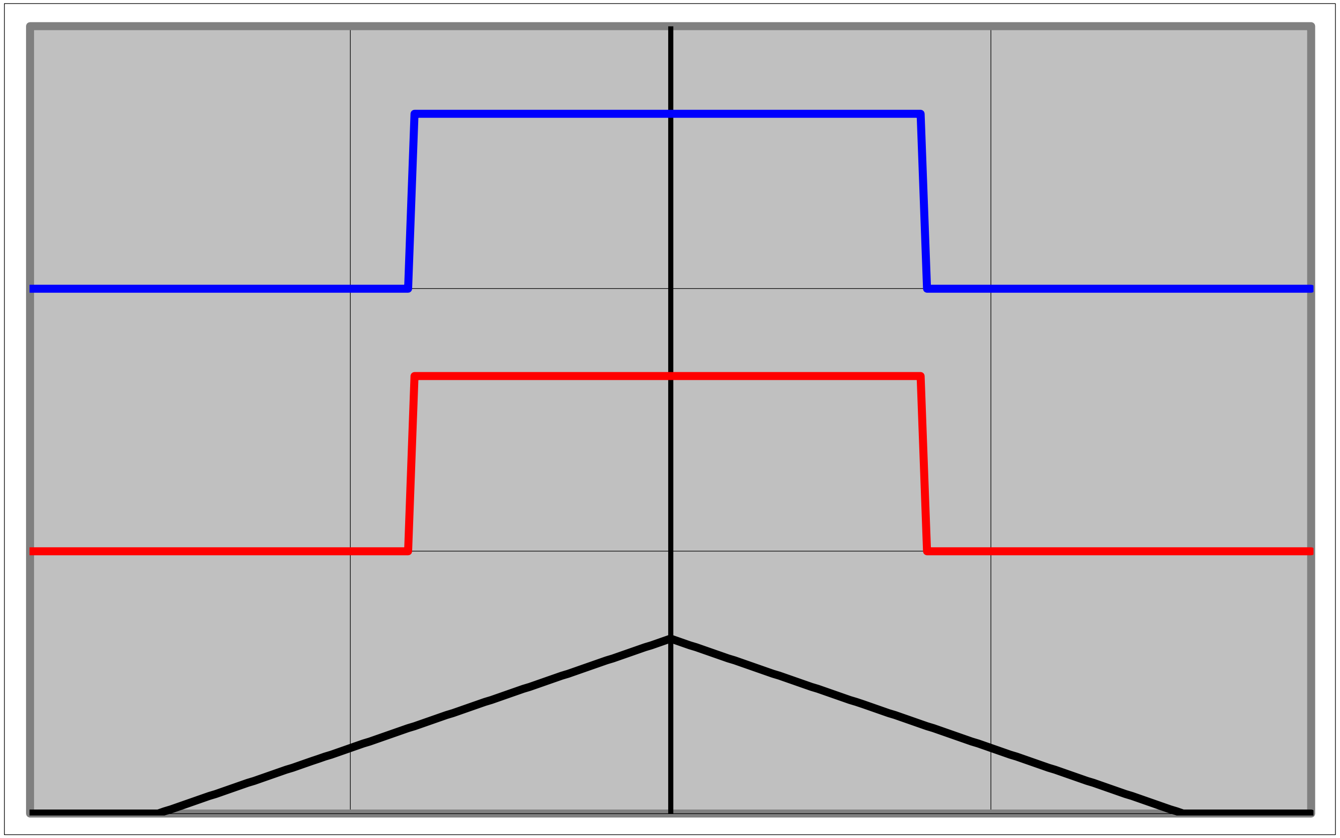
How

- **As a confidence estimate – based on probability**

- **In the same units as the results**

How

Combination of uniform probability distribution functions (PDF)



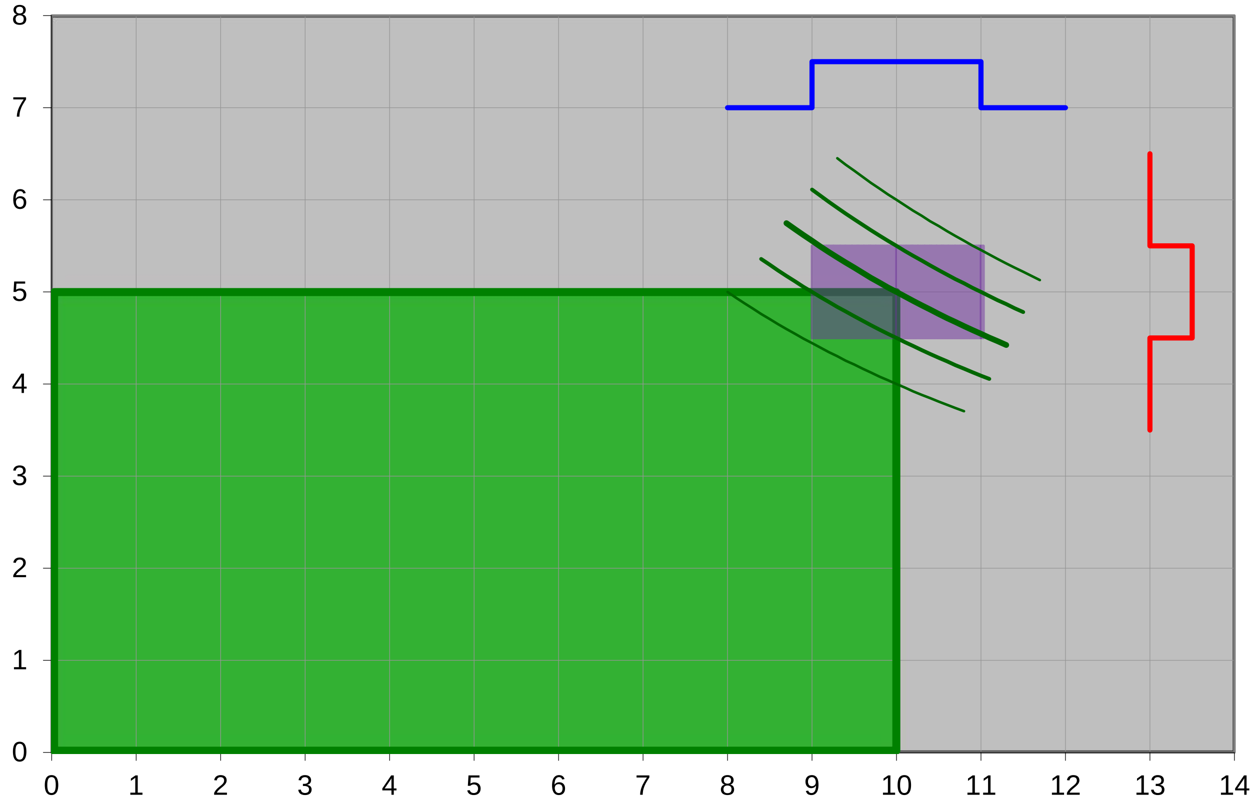
Same as two slits in a monochromator, also known as convolution

How

Calibration limits in a measurement

Find the area of a rectangle

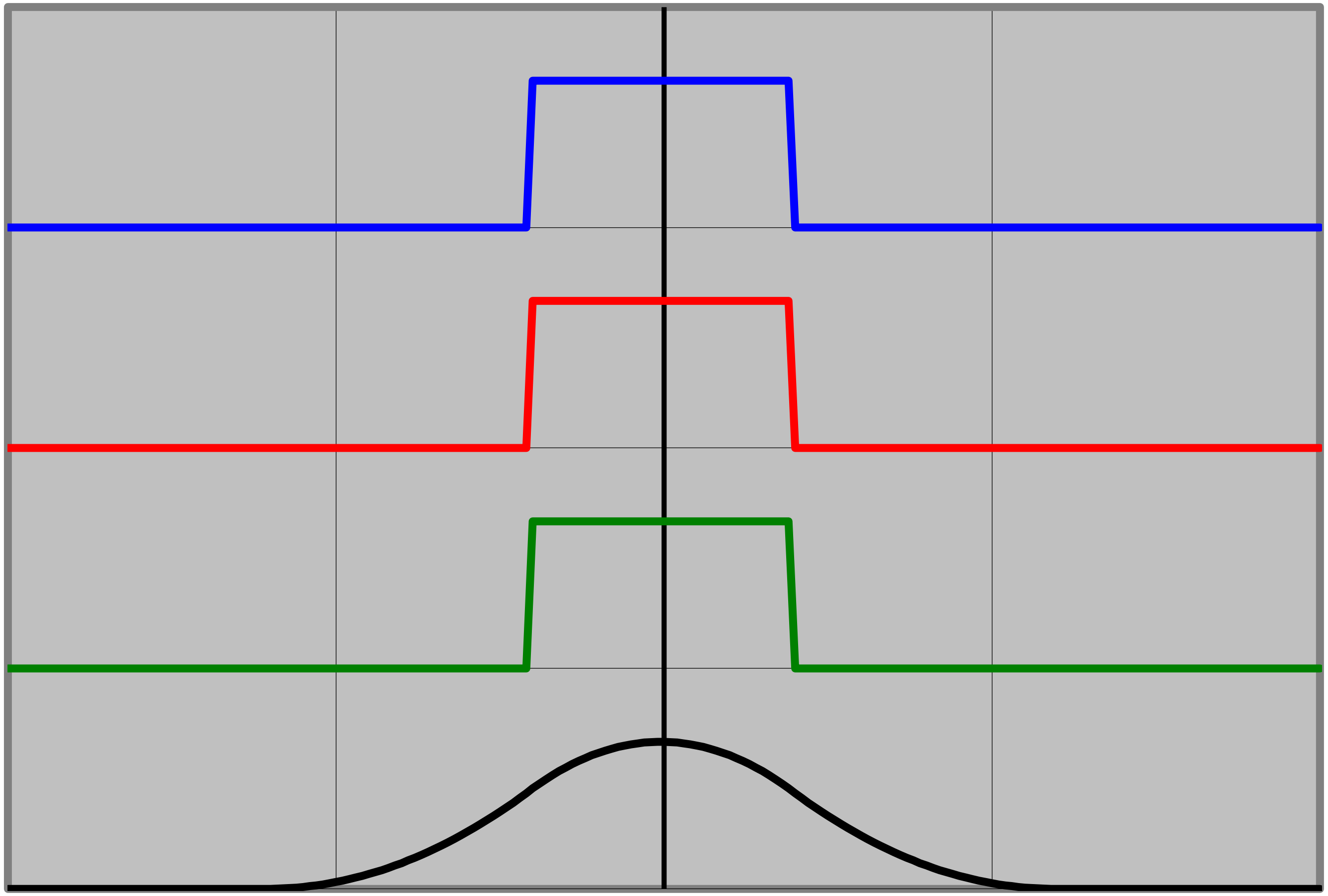
Side measurements limited by calibration to +/- 10 % (length 9 to 11, height 4.5 to 5.5)



Green lines show equal areas for corner position 10 % steps

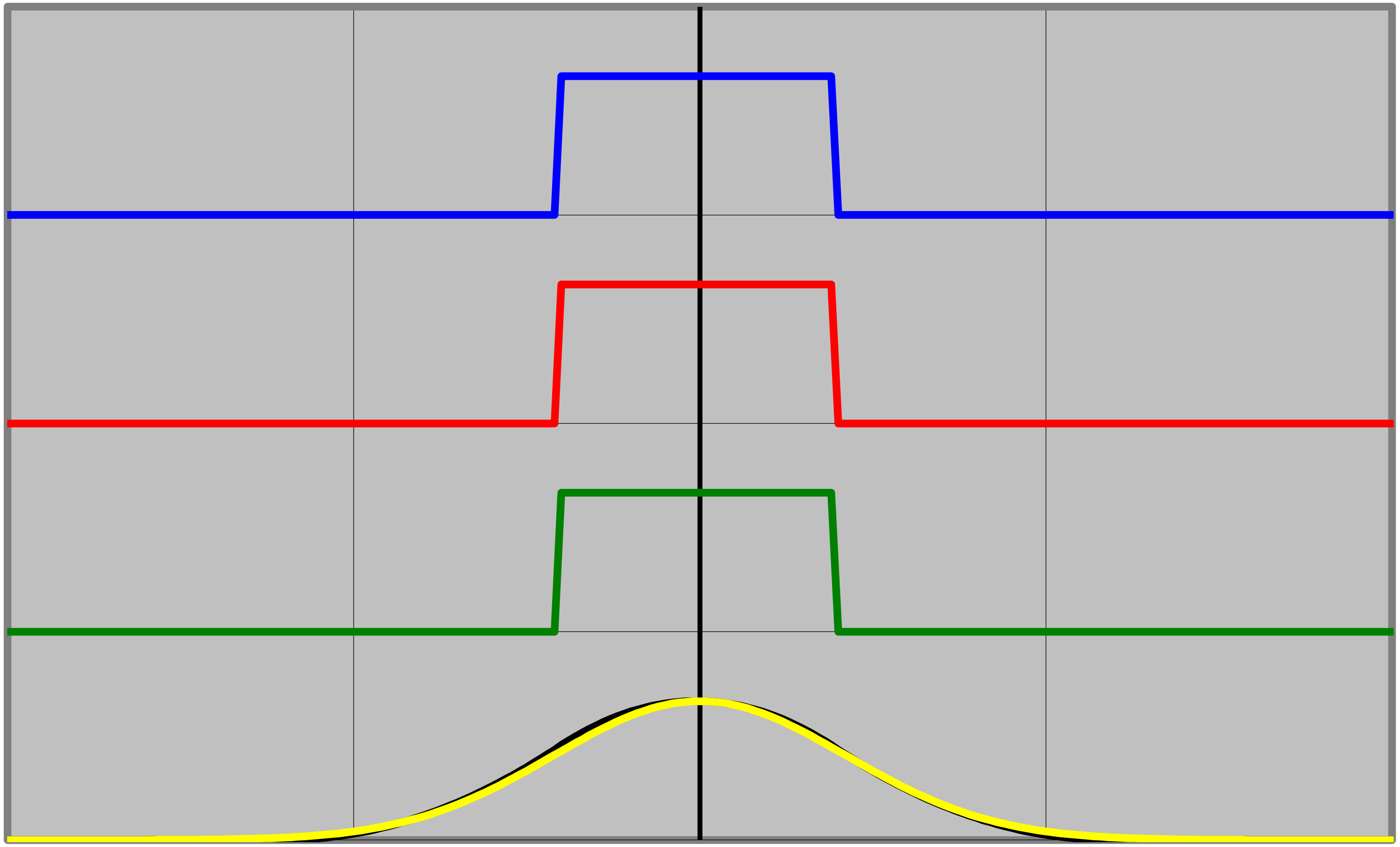
How

Normal PDF is assumed



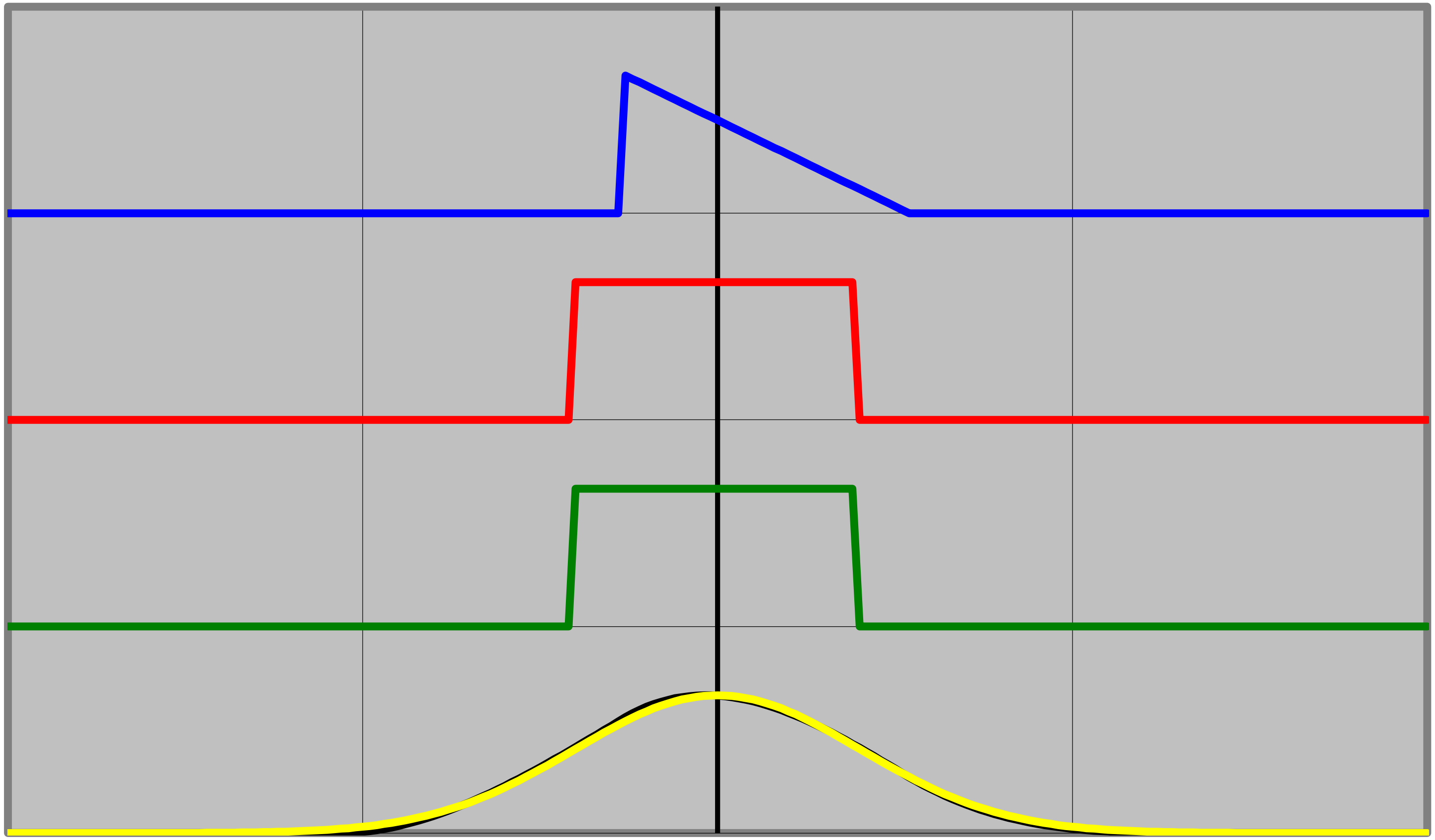
How

Three uniform PDFs combine to give near Normal PDF



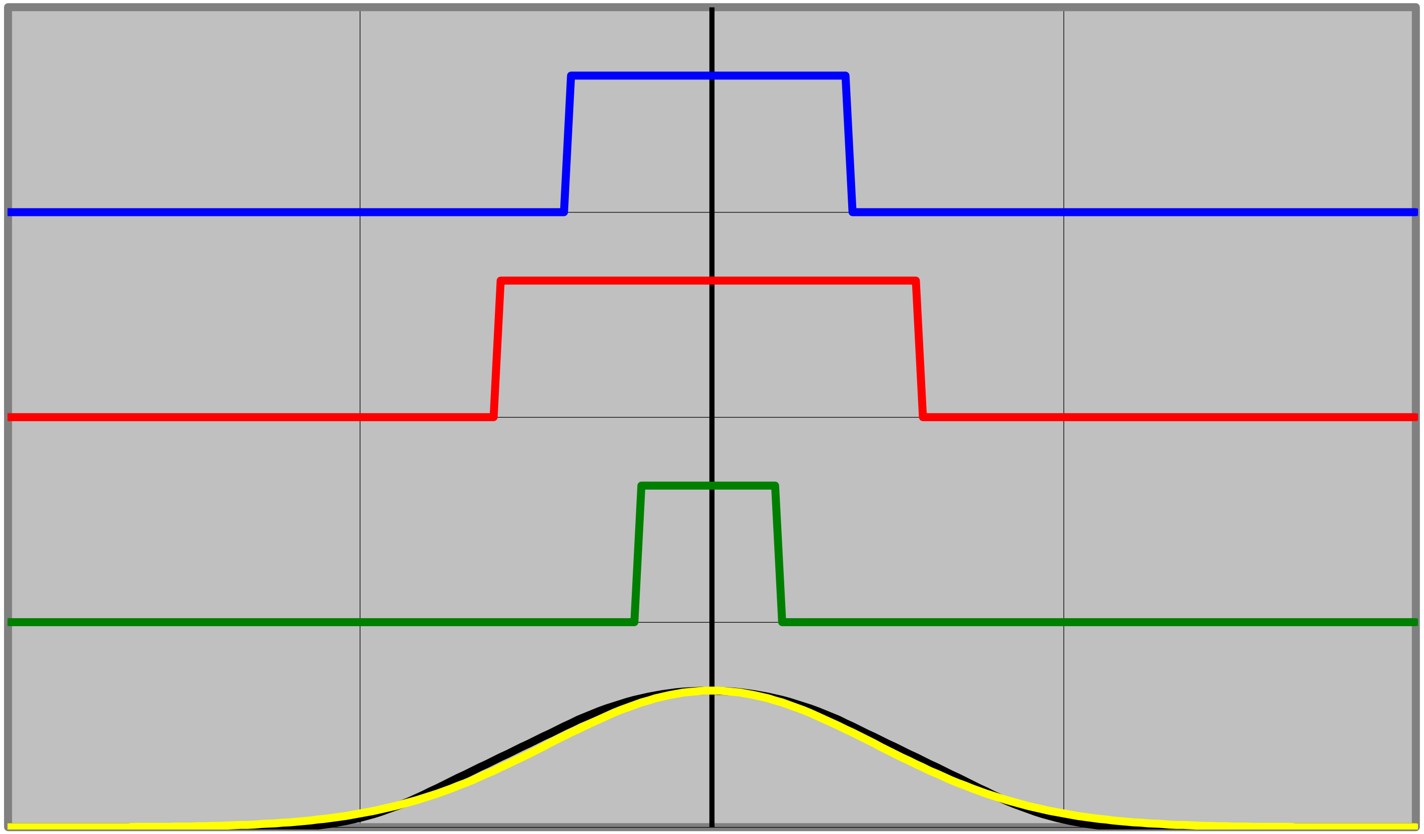
How

Even non uniform PDF combines to near Normal PDF



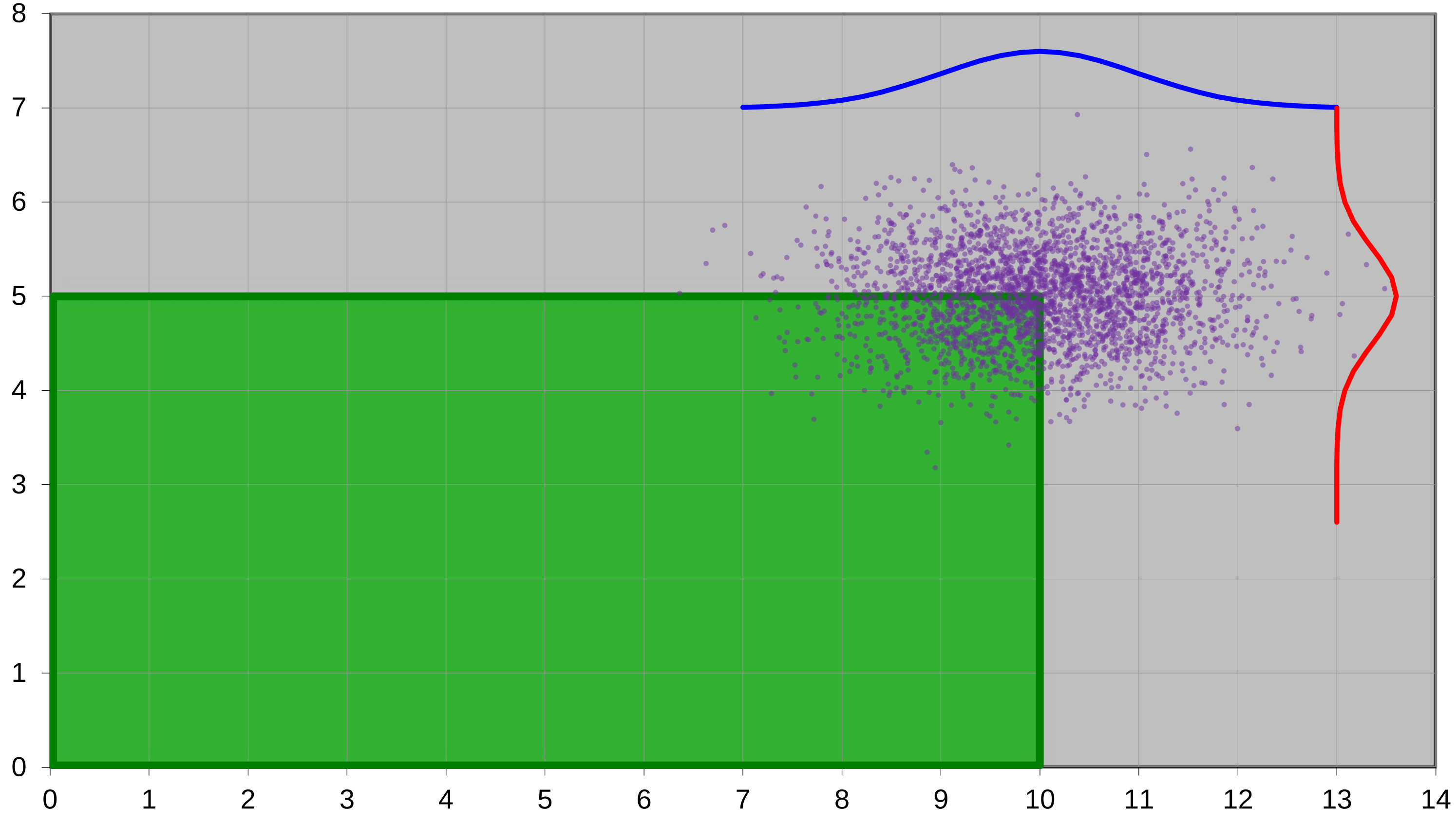
How

Size matters, the larger dominate (in proportion to the square of the size)



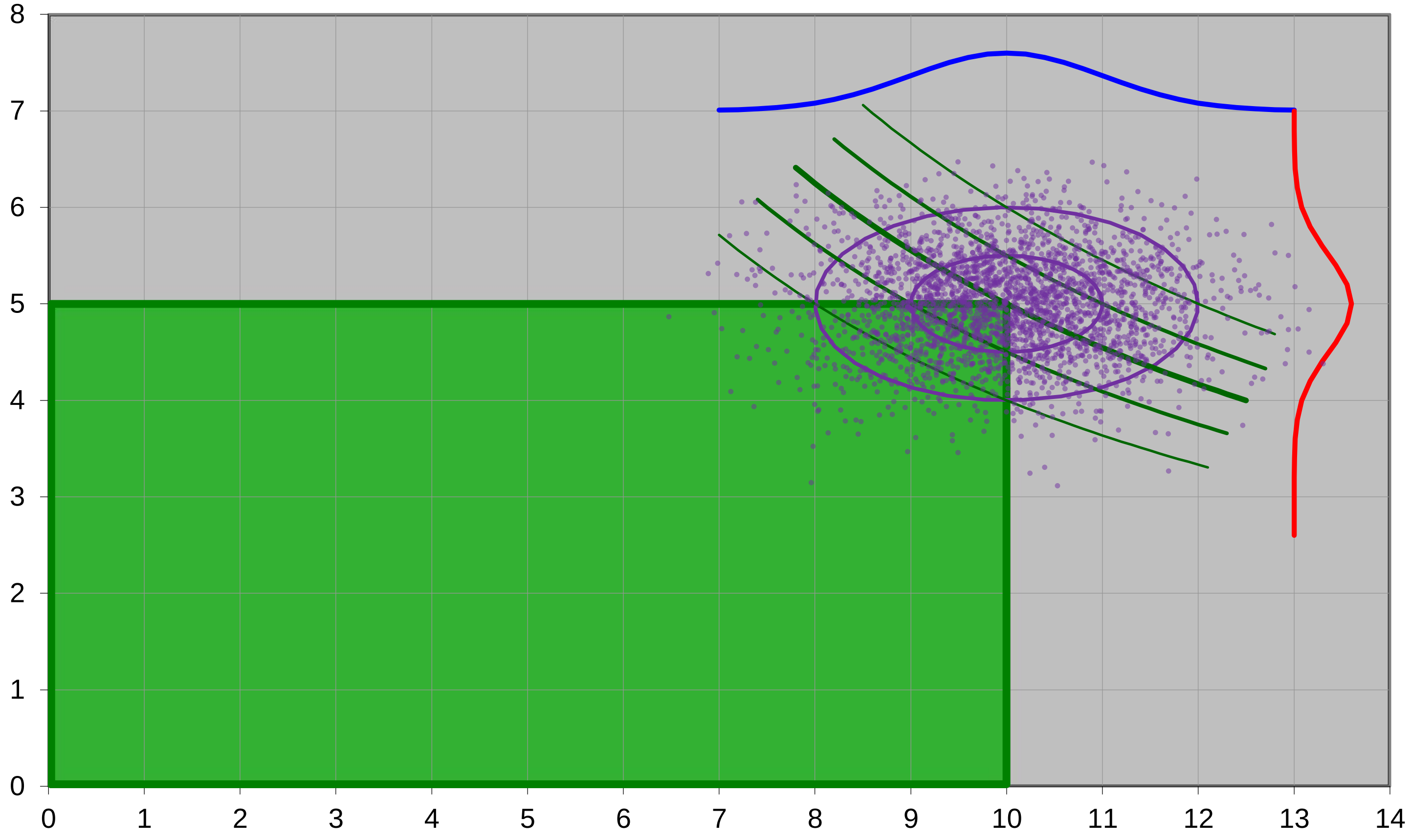
How

Normally distributed uncertainties quantified as Standard Deviation combine as root sum of squared (variances just add)



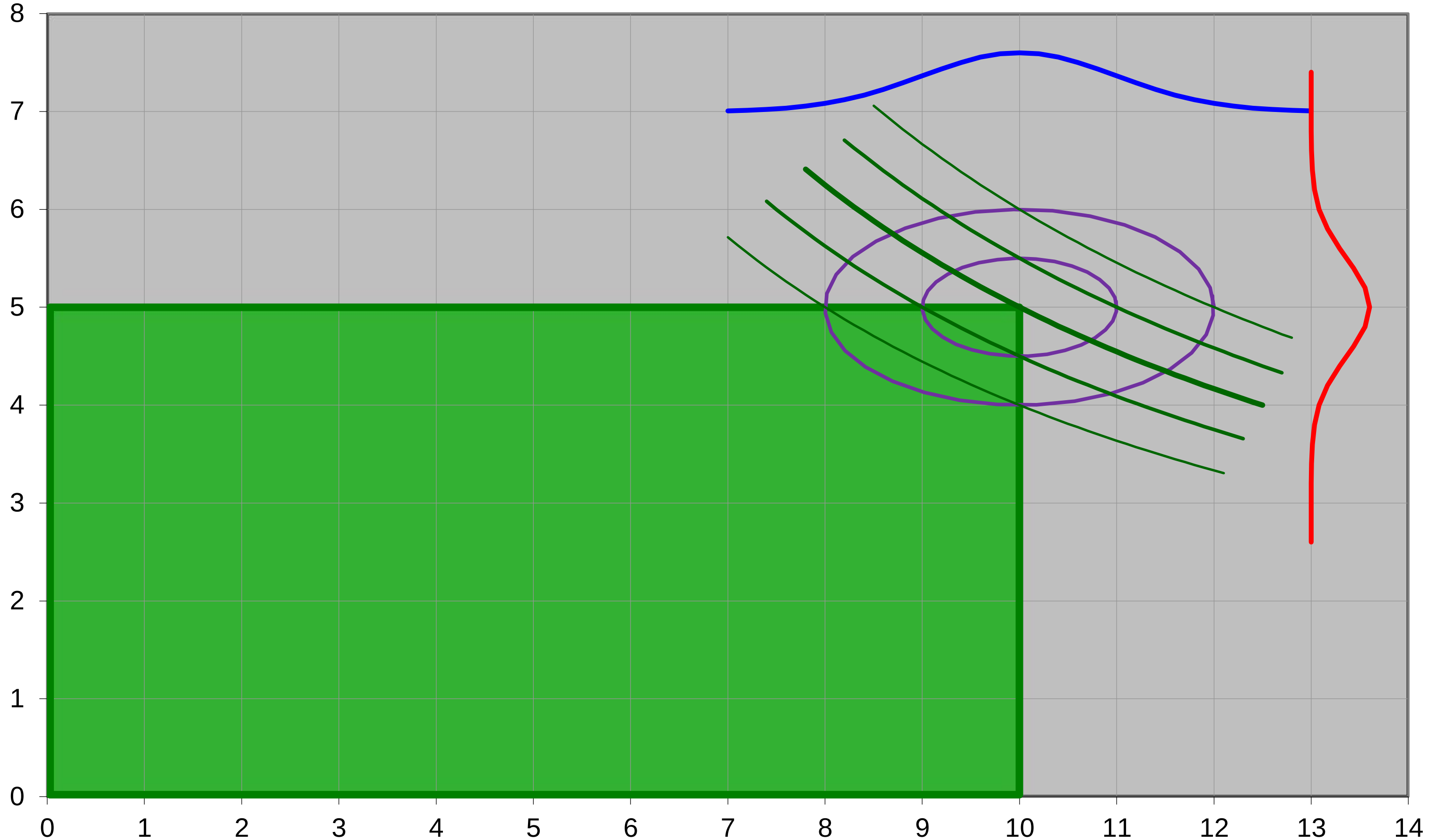
How

1 SD + 1 SD = 1.4 SD : SD^2 + SD^2 = SD^2 : variance + variance = variance



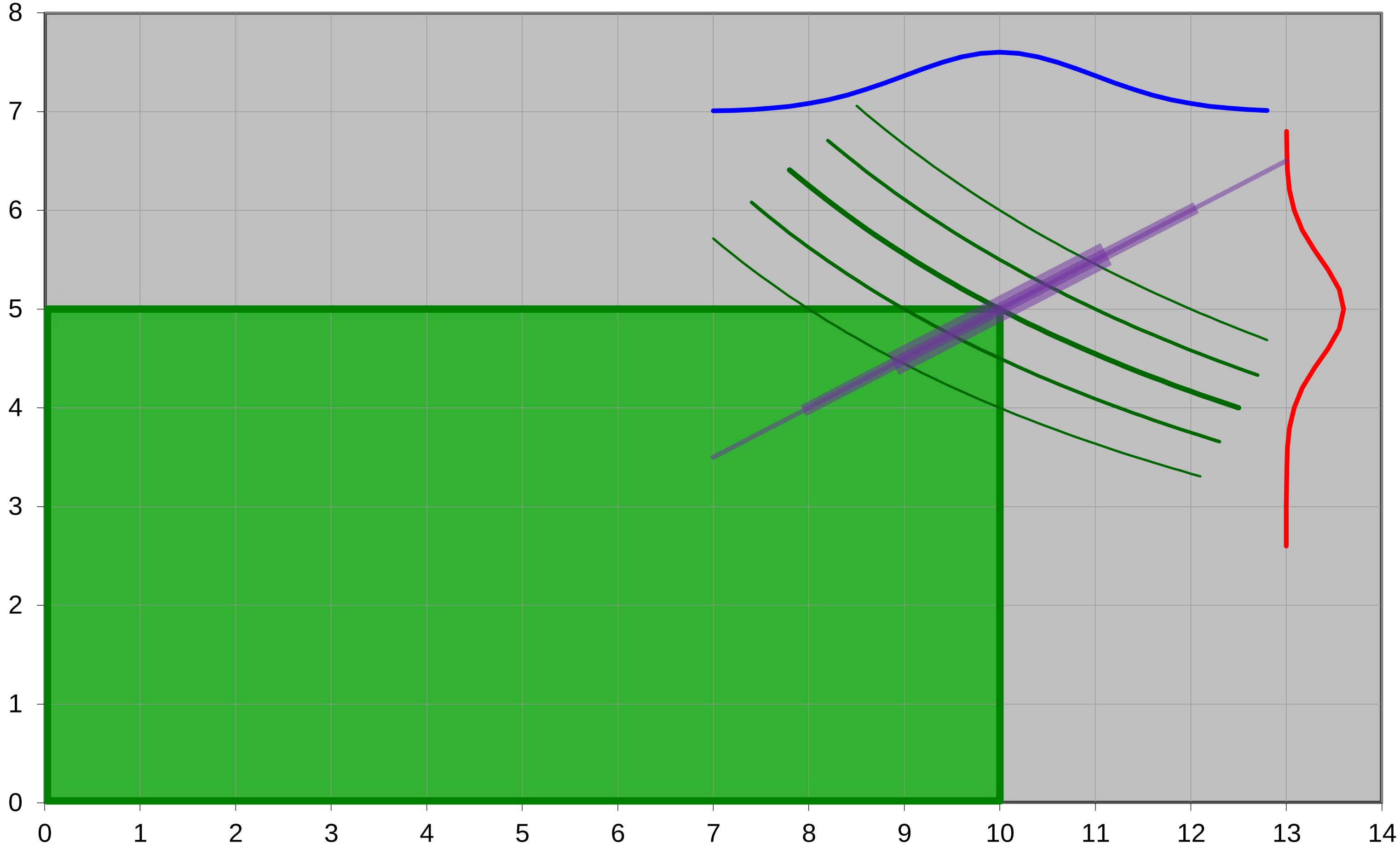
How

1 SD = 10 % Area change 14 %



How

If not independent than add as 1 + 1 = 2

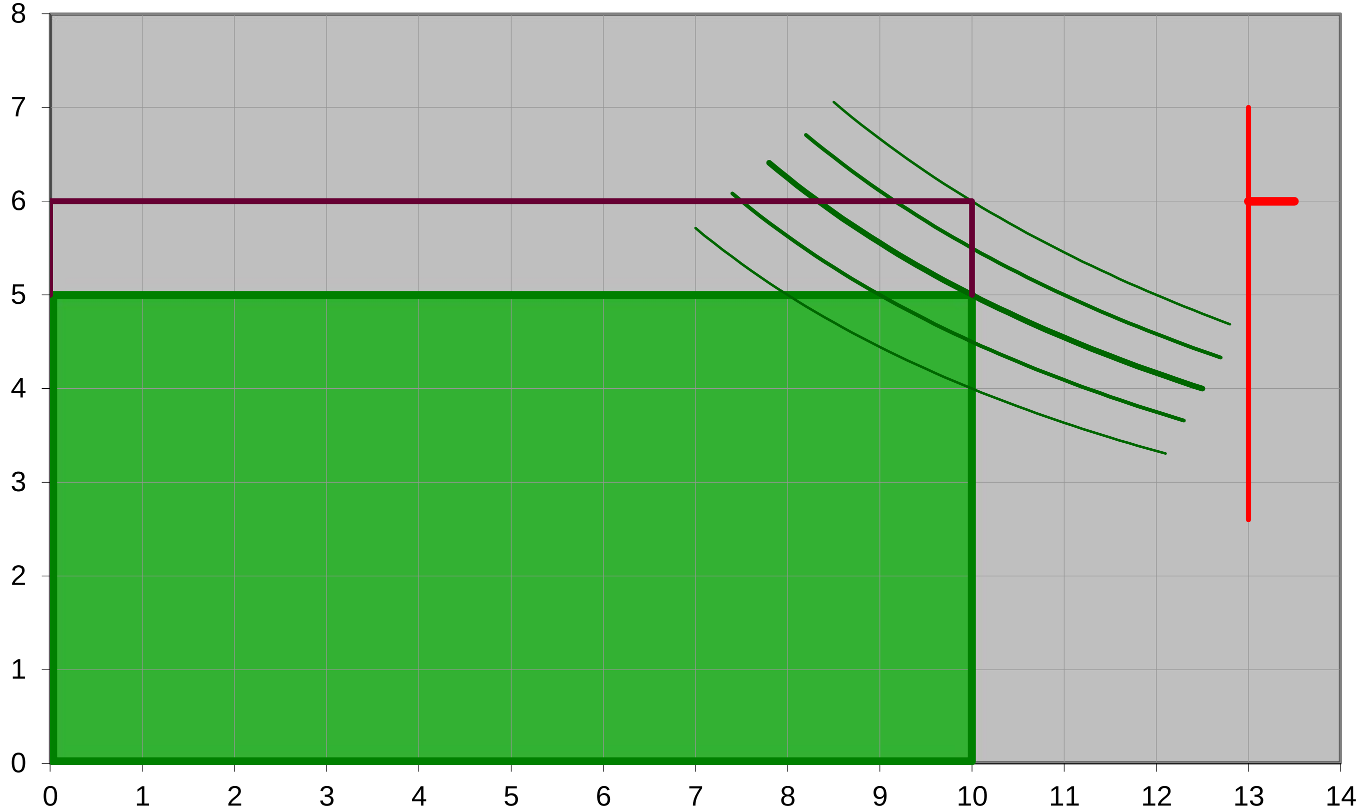


How

Sensitivity of the result to change in the input

Height +1 : Area +10 or Height +20 % : area +20 %

Sensitivity = 10 (length) or 1 unitless (dimensionless)

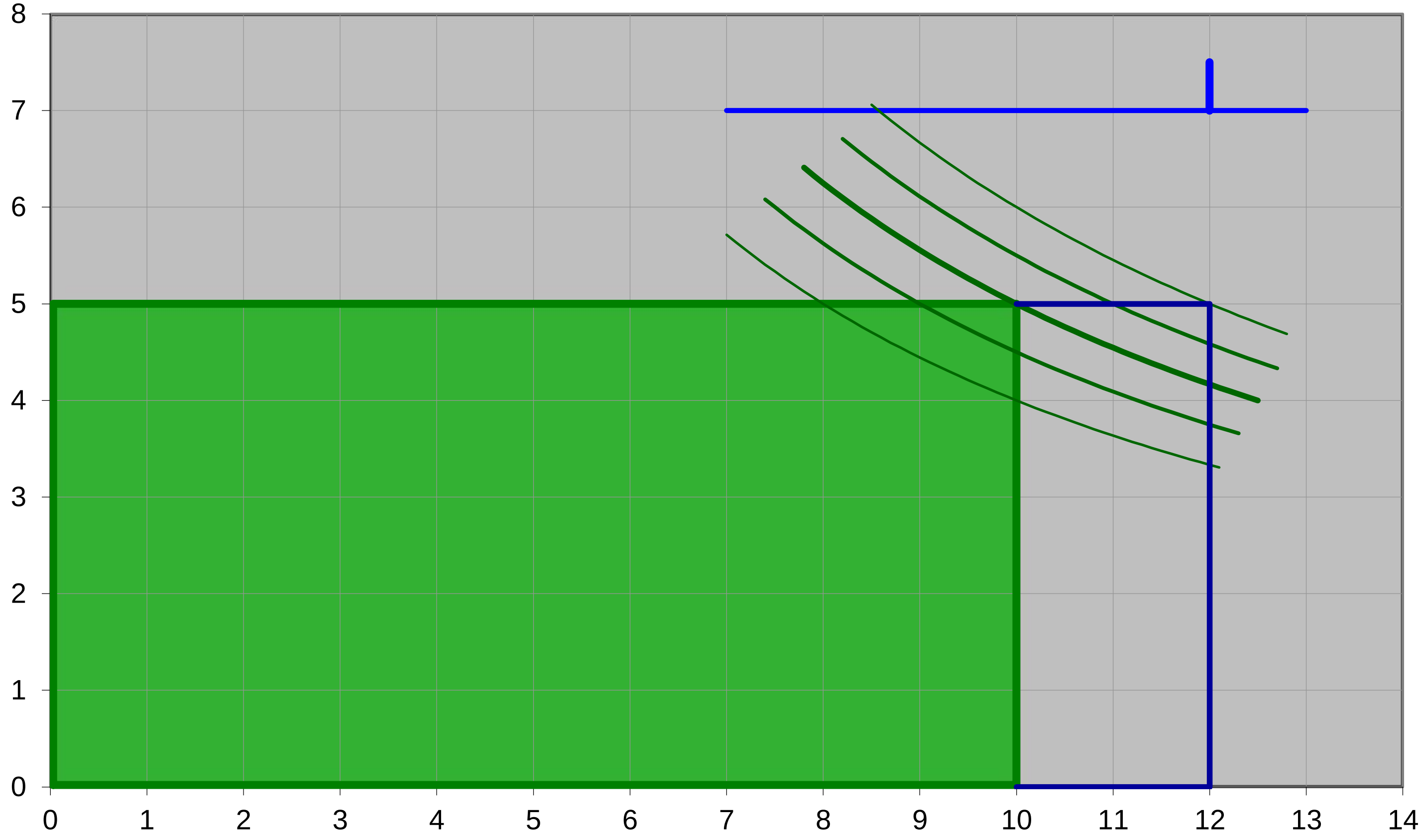


How

Sensitivity of the result to change in the input

Length +2 : Area +10 or Height +20 % : area +20 %

Sensitivity = 5 (height) or 1 unitless (dimensionless)



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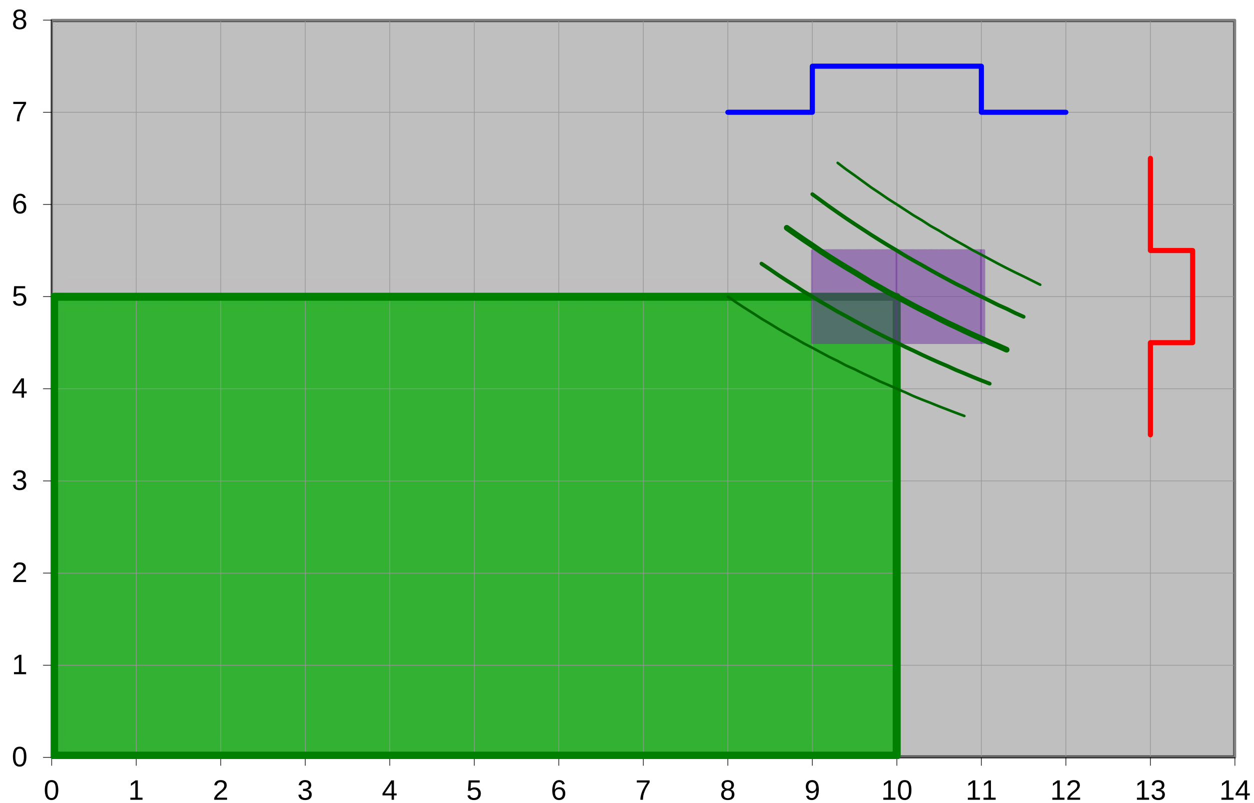

Green lines show equal areas for corner position 10 % steps

How

Measurement Equation

area = length · height

50 = 10 · 5

Uncertainty Equation

$$u_c^2(y) = \sum_{i=1}^{N} \left(\frac{\partial f}{\partial x_i} \right)^2 u^2(x_i) \qquad \text{equ.}10$$

$$u_{area}{}^2 = \left(u_{length} \bullet \frac{\partial_{area}}{\partial_{length}} \right)^2 + \left(u_{height} \bullet \frac{\partial_{area}}{\partial_{height}} \right)^2$$

$$7^2 = (1 \cdot 5)^2 + (0.5 \cdot 10)^2$$

How

Measurement Equation

area = length · height

50 = 10 · 5

Uncertainty Equation

$$\left(\frac{u_{area}}{area}\right)^2 = \left(\frac{u_{length}}{length}\right)^2 + \left(\frac{u_{height}}{height}\right)^2$$

$14\ \%^{2} = 10\ \%^{2} + 10\ \%^{2}$

How

Combining uncertainties is associative and commutative

Input can be combined in any order or combination

Uncertainties are transferred

in from calibration or standards and passed out to users

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How

My three steps

1) Determine **all** inputs or influences that can change the result.

2) Develop a measurement equation and determine sensitivities

3) Do the math

    Combine

    Expand

    Report

How

1) All inputs

What does or could influence your results?

- Include everything
  - if it is not important then the numbers will show it
  - " *it can't be significant!* " may hide a major problem
- How does each vary? – Determine the uncertainty of the input
  - Quantify as a value equal to one standard deviation
    - Formula for non-normal PDFs
  - Measure or uses other means (type A - type B)

How

1) Example inputs

Measurement of length

- Calibration of scale
- Variation in use
- Bias in use
- Temperature
- Humidity
- Gravity
- Magnetic fields
- …

2) Measurement Equation

Result = $f(x_1, x_2, x_3, \ldots)$

Often is a product of terms such as

- length time width
- Voltage times 1/resistance times voltage times time

  $$[(\text{voltage} / \text{resistance}) \cdot \text{voltage}] \cdot \text{time}$$

- Terms for minor influences are often (1 + error)

For relative analysis, terms have a sensitivity of 1

How

3) Do the math

- Combine the terms as absolute or relative including the sensitivities.
- Look for correlations and add correlations terms where needed
- Convert from standard deviation to a confidence of 95 %

  If the degrees of freedom are low then multiply by a factor greater than two to get your 95 % coverage

Important issues properly addressed

- Correlations
- Monte Carlo method for combining inputs and treating sensitivities

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Discussion Starters

- **Is it acceptable to pad a statement of uncertainty?**
  - Can I skip correction just add it to uncertainty?

- **Is variations in the DUT part of the uncertainty?**

- **For turnkey systems can the supplier (designer/implementer) give you an uncertainty?**