Rolf Bergman
Consulting

Evaluating Luminous Uncertainty given Spectral Power Uncertainty

by
Rolf S. Bergman

Content of Presentation

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- Application of GUM Process
  - Defining Equation or Model
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Introduction

- Integrating spheres are normally used to measure lumen output of a lamp
- Calibration of integrating sphere is done using total spectral flux standard lamp
  - Standard lamp calibration certificate provide values for spectral power flux at given wavelengths and values for the uncertainty of the flux at each of those wavelengths.
- Lumens are calculated from test lamp spectral flux
- **How does the uncertainty in the spectral flux get translated to lumen uncertainty?**

Lumen Equation

- The lumen value is the sum (integral) of the spectral power, S_i, over the wavelength region of interest, weighted by the photopic efficacy, V_i, i.e.,

$$\Phi = 683 \sum_{i=1}^{N} V_i(\lambda) \cdot S_i(\lambda) \cdot \Delta\lambda$$

Lumen Uncertainty

- The only term on the right-hand-side of the lumen equation that has uncertainty is the spectral flux values at a given wavelength, λ_i.
- Thus the uncertainty in the lumens is a function of the uncertainty in the spectral flux, i.e.,

$$u(\Phi) = f[u(S_i)]$$

Combined Uncertainty

- The Combined Uncertainty when correlation is defined as the square root of the product of the sum of the sensitivity coefficient, $\partial\Phi/\partial x_i$ at that spectral value, x_i, times the sensitivity coefficient at every other spectral value, times the covariance associated with each spectral value pair, $u(x_i, x_j)$. This can be written as:

$$u^2(\Phi) = \sum_{i=1}^{N} \left(\frac{\partial\Phi}{\partial S_i}\right)^2 u^2(S_i) + 2\sum_{i=1}^{N-1} \frac{\partial\Phi}{\partial S_i} \cdot u(S_i) \sum_{j=i+1}^{N} \frac{\partial\Phi}{\partial S_j} \cdot u(S_j) \cdot r(S_i, S_j)$$

where $r(S_i,S_j)$ is the correlation coefficient.

Sensitivity coefficients

- In the last equation we noted that the sensitivity coefficient was the partial derivative of the lumen value with respect to the variables.
- There is only one variable with uncertainty in the lumen equation, that of S_i. Thus:

$$\frac{\partial \Phi}{\partial S_i} = \frac{\partial\left[683 \sum_{i=1}^{N} V_i(\lambda) \cdot S_i(\lambda) \cdot \Delta\lambda\right]}{\partial S_i} = 683 \cdot V_i \cdot \Delta\lambda$$

Combined Uncertainty for Lumens

- Inserting the values of the Sensitivity Coefficient into the combined uncertainty we obtain:

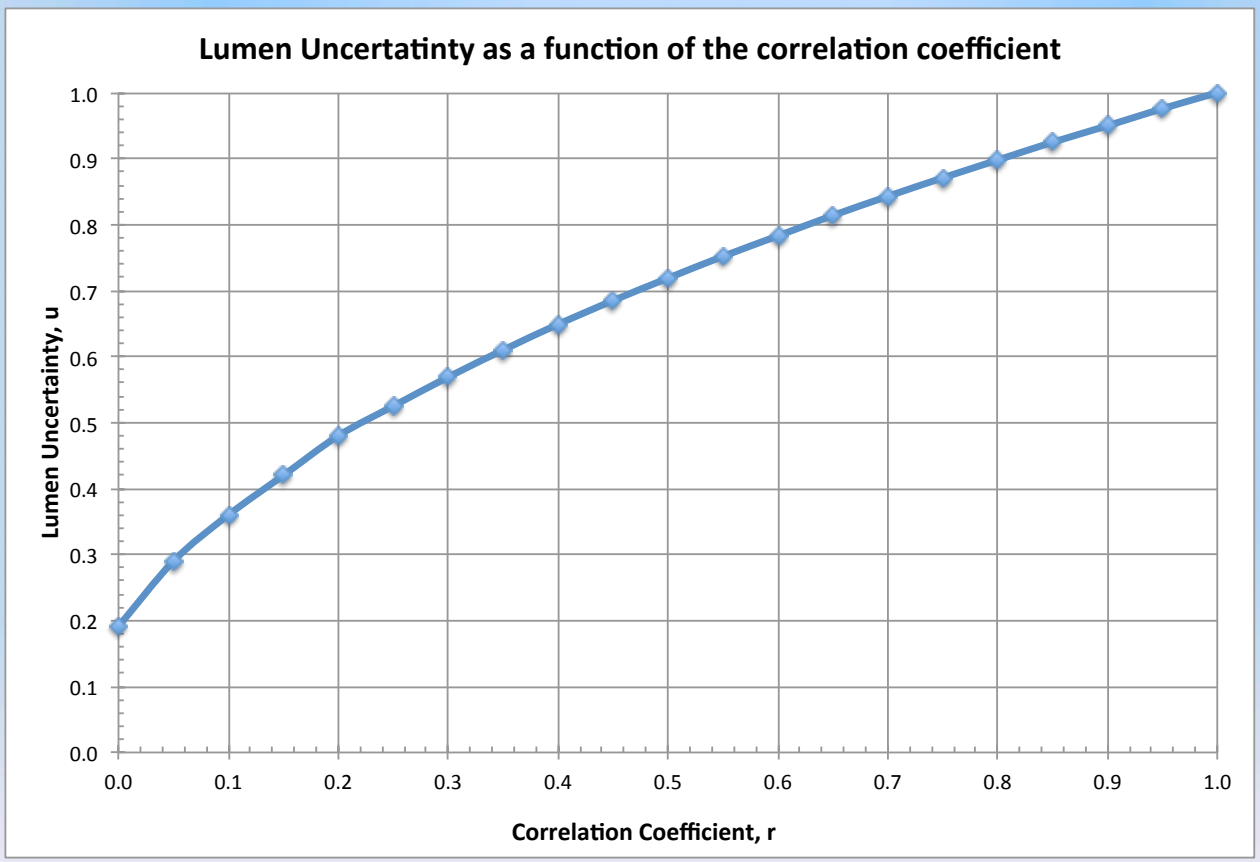
$$u^2(\Phi) = (683 \cdot \Delta\lambda)^2 \left(\sum_{i=1}^{N} V_i{}^2 \cdot (hS_i)^2 + 2 \sum_{i=1}^{N-1} V_i \cdot h_i S_i \cdot \sum_{j=i+1}^{N} V_j \cdot h_j S_j \cdot r \right)$$

- where h_i is the fractional uncertainty in the spectral value, S_i.

Evaluation of Lumen Uncertainty

- Combined uncertainty equation is evaluated using an EXCEL spreadsheet.
- Assumptions:
  - A NIST halogen standard lamp with given spectral flux values, S_i, every 5 nm between 380 and 780 nm.
  - The relative spectral uncertainty is accounted for by blue, green and red values; $h_i = h_1$, h_2 and h_3.
  - The correlation coefficient is not known but we will evaluate the effect of various values of r on the value of the lumen uncertainty, $u(\Phi)$.

Lumen Uncertainty



Conclusions

- Wavelength Integration:
  - Ignoring the spectral uncertainty except in the central visible region where Vi > 0.1 (about 175 nm band) causes negligible errors in the lumen uncertainty
- Correlation Coefficient:
  - What is a reasonable correlation coefficient between spectral flux values at various wavelengths? r=0.5?
  - If so the reduction in the lumen uncertainty due to multiple values is small, about 21 %
  - As a general rule it is reasonable to assume that the lumen uncertainty is equal to the spectral uncertainty for spectral power in the neighborhood of 560 nm.