

Practical Approaches to Uncertainty Analysis for Light Measurement Systems Using Integrating Spheres and Spectroradiometers

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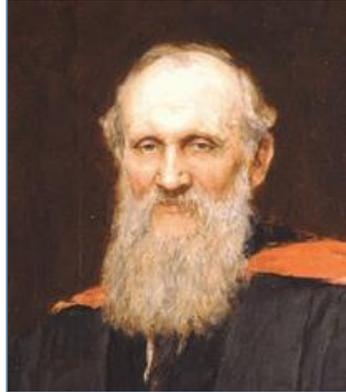


Outline

- ▶ Background
- ▶ Uncertainty Overview
- ▶ The Integrating Sphere / Spectroradiometer System
- ▶ Uncertainty Contributions
- ▶ Summary



Uncertainty Overview



“I often say that when you can measure what you are speaking about, and express it in numbers, you know something about it.”

- Lord Kelvin

- ▶ Why?
 - ▶ Because we have to.
 - ▶ IEC/ISO17025
 - ▶ NVLAP HB-150
 - ▶ Because we need to.
 - ▶ Sets a level playing field.
 - ▶ Identifies areas of concern
 - ▶ Can identify changes in process and gauge quality

Uncertainty Overview

▶ Case Study References:

- ▶ J. Leland, “Uncertainty Analysis for an Integrating Sphere Photometer - A Case Study,” CORM 2014
- ▶ A. Jackson, “Case Study: Calibration Uncertainty Analysis for an Integrating Sphere Spectroradiometer,” CORM 2014
- ▶ R. Bergman, M. Paget, E. Richman, “CALiPER Exploration Study: Accounting for Uncertainty in Lumen Measurements,” USDOE, PNNL-20320. March 2011

▶ Uncertainty Methods:

- ▶ D. Gross, “Interpolation, Correlation and NIST Standards,” CORM 2014

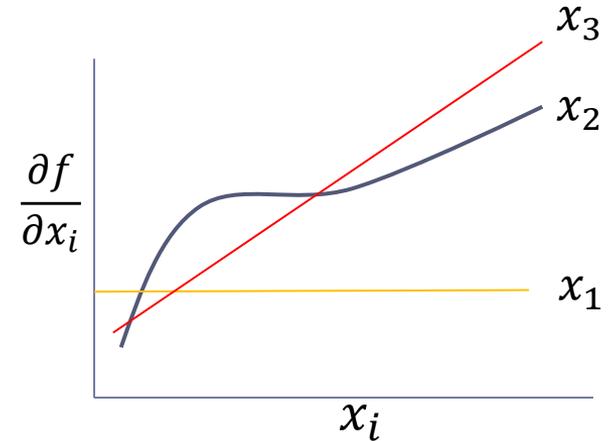
Uncertainty Overview

1) Define the fundamental equation: $y = f(x_1, x_2, x_3, \dots, x_n)$
(measurand)

2) Calculate the Sensitivities: $C_{x_i} = \frac{\partial f}{\partial x_i}$

3) Determine the Uncertainties: $u(x_i)$

4) Calculate the Coverage Factor: $k_{0.95}$



Law of
Propagation of
Uncertainties:

Sensitivity Contribution;
add in quadrature (RSS)

Correlation
Term

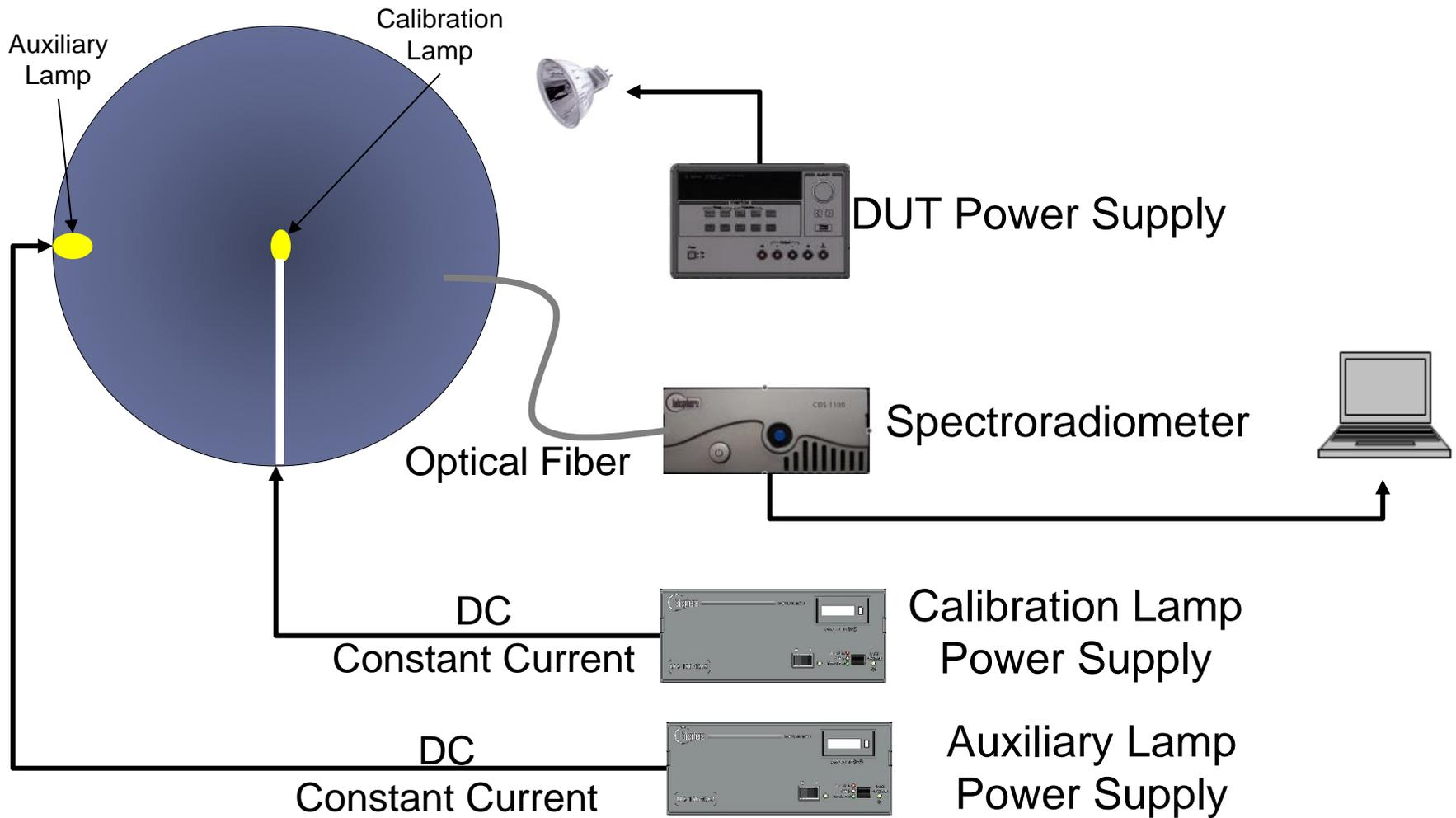
Combined Standard Uncertainty \rightarrow

$$u_c^2(y) = \sum_{i=1}^n \left(\frac{\partial f}{\partial x_i} \right)^2 u^2(x_i) + 2 \sum_{i=1}^{n-1} \sum_{j=i+1}^n \frac{\partial f}{\partial x_i} \frac{\partial f}{\partial x_j} u(x_i, x_j)$$

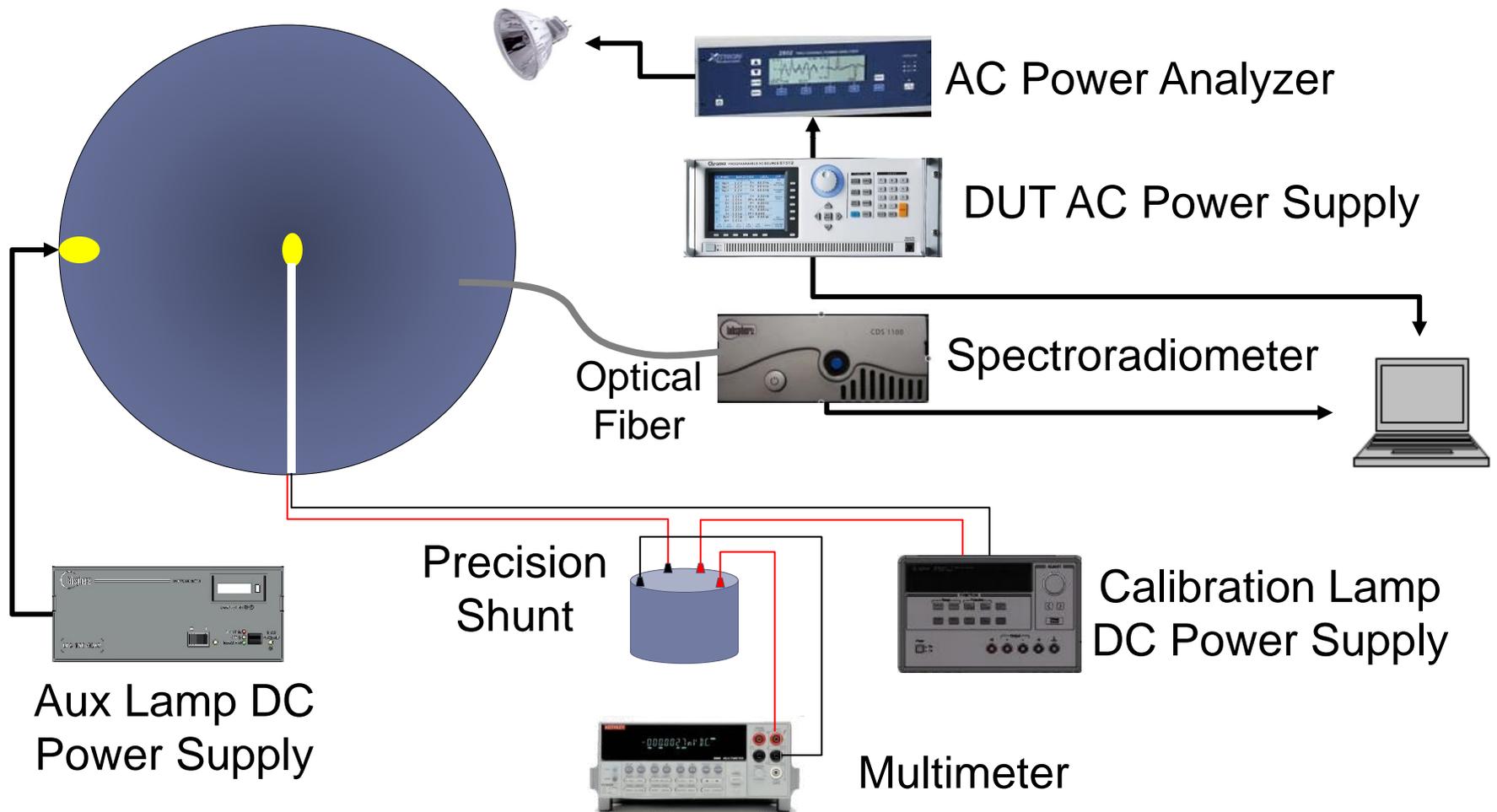
Uncertainty Overview

- ▶ How do we get the uncertainty estimates for each term?
 - ▶ Measure it – acquire statistical data
 - ▶ Estimate it based on literature
 - ▶ Calculate it numerically
 - ▶ Analytical/Empirical model
 - ▶ Product data sheets

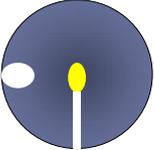
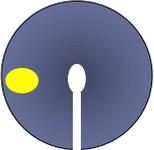
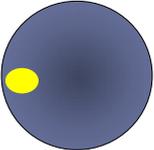
Integrating Sphere / Spectroradiometer: System 1



Integrating Sphere / Spectroradiometer: System 2 – AC with DC Shunt Resistor



Calibration Procedure: 4 Measurements

	Configuration	Measurement	
1.	Reference Standard On	$y_R(\lambda)$	
2.	Aux. Lamp On, Ref Lamp Off	$y_{xR}(\lambda)$	
3.	Aux Lamp On, Empty Sphere	$y_{xE}(\lambda)$	
4.	Aux Lamp On, DUT in place	$y_{xDUT}(\lambda)$	
5.	DUT On (Aux off)	$y_{DUT}(\lambda)$	

Measurement Details

- ▶ A “dark” scan is taken prior to each measurement to remove the baseline offset from the scan.
 - ▶ e.g., for the Ref. Lamp: $y_R(\lambda) - y_{R_d}(\lambda)$
- ▶ All Scans are wavelength dependent.
 - ▶ Spectral range depends on Ref. Standard and Spectrometer
- ▶ Spectrometer Manipulations:
 - ▶ Pixel Interpolation to get integer wavelength values
 - ▶ Stray Light Correction
 - ▶ “Counts” conversion to engineering units
- ▶ Applying Calibration Certificate Values
 - ▶ Interpolation

Uncertainty Overview: Measuring Spectral Flux

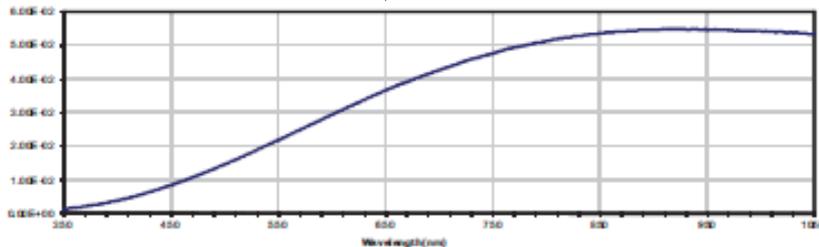
$$\phi_{DUT}(\lambda) = \frac{(y_{DUT}(\lambda) - y_{DUT_d}(\lambda))}{(y_R(\lambda) - y_{R_d}(\lambda))} \cdot \phi_{R,\lambda}(\lambda) \cdot \frac{L_e(\lambda, T)}{L_e(\lambda, T_0)} \cdot \frac{(y_{x_R}(\lambda) - y_{x_{Rd}}(\lambda))}{(y_{x_{DUT}}(\lambda) - y_{x_{DUT_d}}(\lambda))}$$

Scale Factor

Ref. Lamp
Calibration

Spectral
Shift
Correction

Aux Correction



$L_e(\lambda, T_0)$ = Spectral Radiance at T_0

$L_e(\lambda, T)$ = Spectral Radiance at T

R = Ref Scan

d = Dark Scan

x = Aux Scan

DUT = Device Scan

$L_e(\lambda)$ = Spectral Shift

Uncertainty Overview: The Sphere Calibration Coefficient

$$\phi_{DUT}(\lambda) = \frac{(y_{DUT}(\lambda) - y_{DUT_d}(\lambda))}{(y_R(\lambda) - y_{R_d}(\lambda))} \cdot \phi_{R,\lambda}(\lambda) \cdot \frac{L_e(\lambda, T)}{L_e(\lambda, T_0)} \cdot \frac{(y_{x_R}(\lambda) - y_{x_{R_d}}(\lambda))}{(y_{x_{DUT}}(\lambda) - y_{x_{DUT_d}}(\lambda))}$$

Isolate the terms from the calibration of the system, normalize to an empty sphere, include correction terms.

$$C_\phi(\lambda) = \frac{(y_R(\lambda) - y_{R_d}(\lambda))}{\phi_{R,\lambda}(\lambda)} \cdot \frac{L_e(\lambda, T_0)}{L_e(\lambda, T)} \cdot \frac{(y_{x_E}(\lambda) - y_{x_{E_d}}(\lambda))}{(y_{x_R}(\lambda) - y_{x_{R_d}}(\lambda))} \cdot A_R C_{fR}$$

A_R = Aging Factor

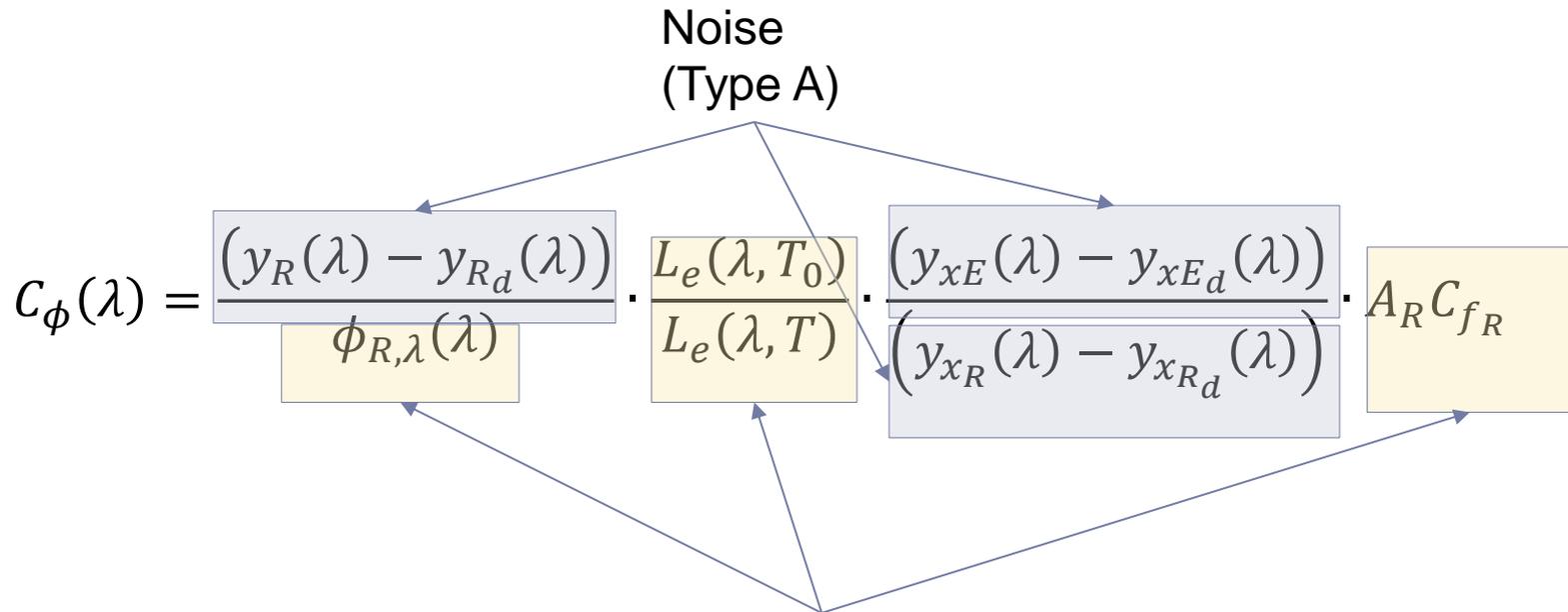
C_{fR} = Additional Contributions

Uncertainty Overview: Using the Sphere Calibration Coefficient

$$\phi_{DUT}(\lambda) = \frac{(y_{DUT}(\lambda) - y_{DUT_d}(\lambda))}{C_{\phi}(\lambda)} \cdot \frac{(y_{xE}(\lambda) - y_{xE_d}(\lambda))}{(y_{DUT_{AUX}}(\lambda) - y_{DUT_{AUX_d}}(\lambda))}$$

Measurement of DUT utilizes an Aux-Empty Scan.

Sources of Uncertainty



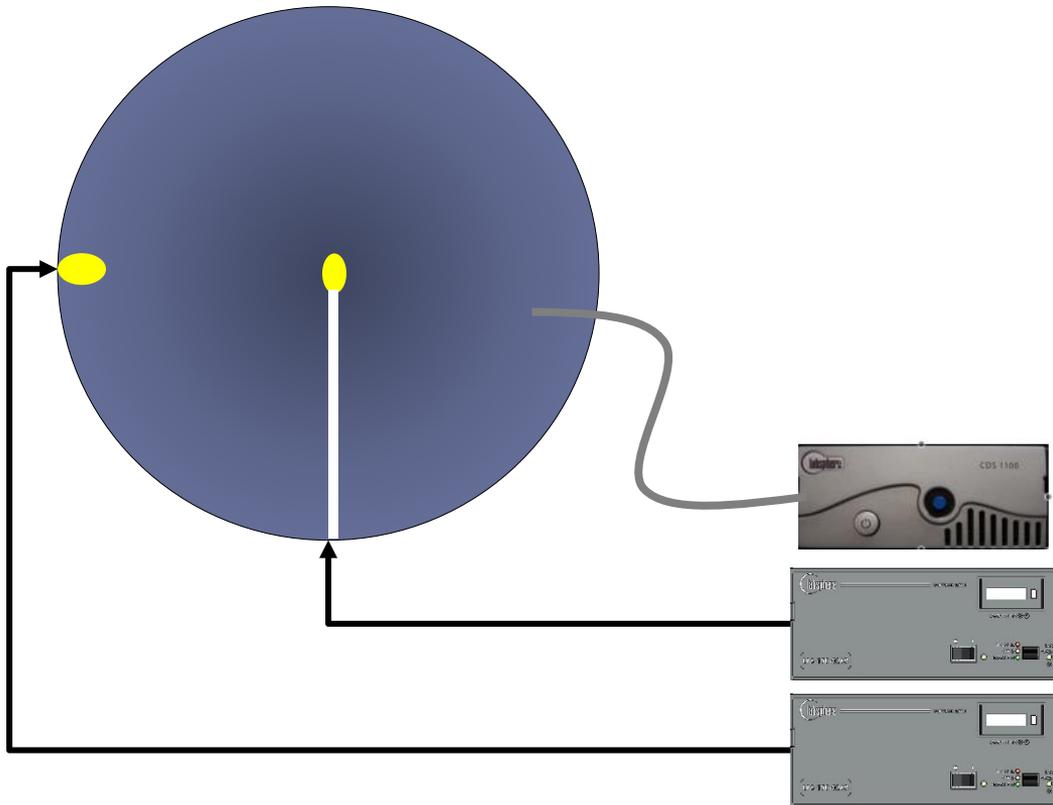
Look at each term separately and identify sources of uncertainty for each term.

Sources of Uncertainty

Variable	Impact	Uncertainty Source
$\phi_{R,\lambda}(\lambda)$ Spectral Flux	Flux	Spectral Resolution Ref. Lamp Uncertainty
$\frac{L_e(\lambda, T_0)}{L_e(\lambda, T)}$	Spectral Shift Total Flux	Lamp Current Ambient Temperature
A_R Aging Factor	Flux	Lamp Decay
C_{fR} Additional Terms	Flux	Spectrometer Stray Light Light Leakage into Sphere Near Field Absorption Sphere Non-uniformity Spectrometer Nonlinearity Ambient Temperature

Uncertainty Contributions: Measured Values

$$C_{\phi}(\lambda) = \frac{(y_R(\lambda) - y_{R_d}(\lambda))}{\phi_{R,\lambda}(\lambda)} \cdot \frac{L_e(\lambda, T_0)}{L_e(\lambda, T)} \cdot \frac{(y_{xE}(\lambda) - y_{xE_d}(\lambda))}{(y_{xR}(\lambda) - y_{xR_d}(\lambda))} \cdot A_R C_{fR}$$



1. Take “N” scans (N=3 to 10)
2. Calculate the mean
3. Calculate std. dev.
4. $u(y(\lambda) - y_d(\lambda)) = \frac{Std.Dev.}{Mean}$

Measuring multiple scans of the reference lamp includes:

- Spectrometer noise
- Power Supply noise
- Dark Subtraction

Uncertainty Contributions: Ref. Lamp Calibration Certificate, $\phi_{R,\lambda}(\lambda)$

$$C_{\phi}(\lambda) = \frac{(y_R(\lambda) - y_{R_d}(\lambda))}{\phi_{R,\lambda}(\lambda)} \cdot \frac{L_e(\lambda, T_0)}{L_e(\lambda, T)} \cdot \frac{(y_{x_E}(\lambda) - y_{x_{E_d}}(\lambda))}{(y_{x_R}(\lambda) - y_{x_{R_d}}(\lambda))} \cdot A_R C_{f_R}$$

Uncertainty from Interpolation and Wavelength Accuracy

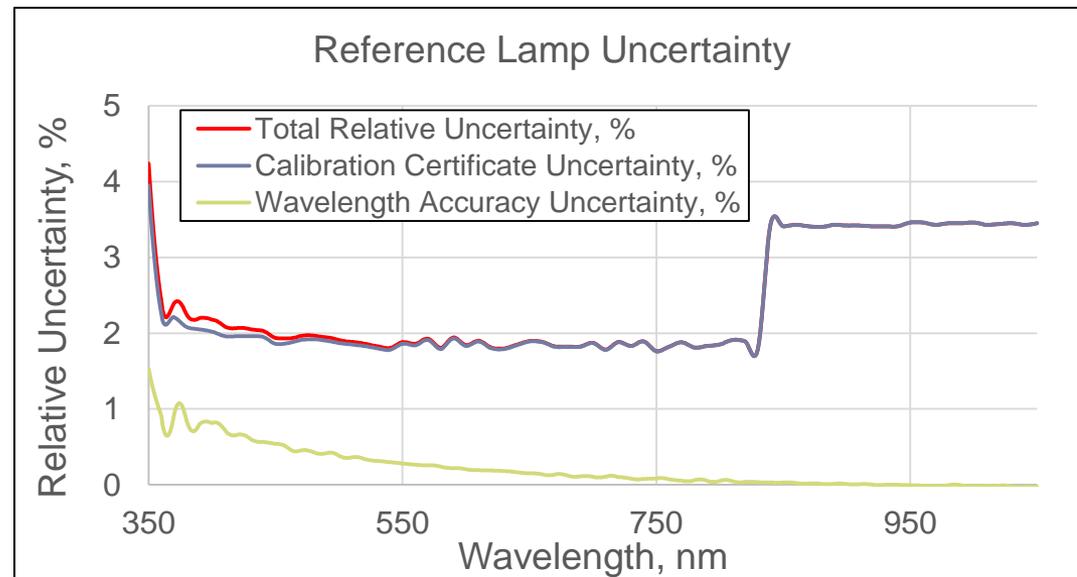
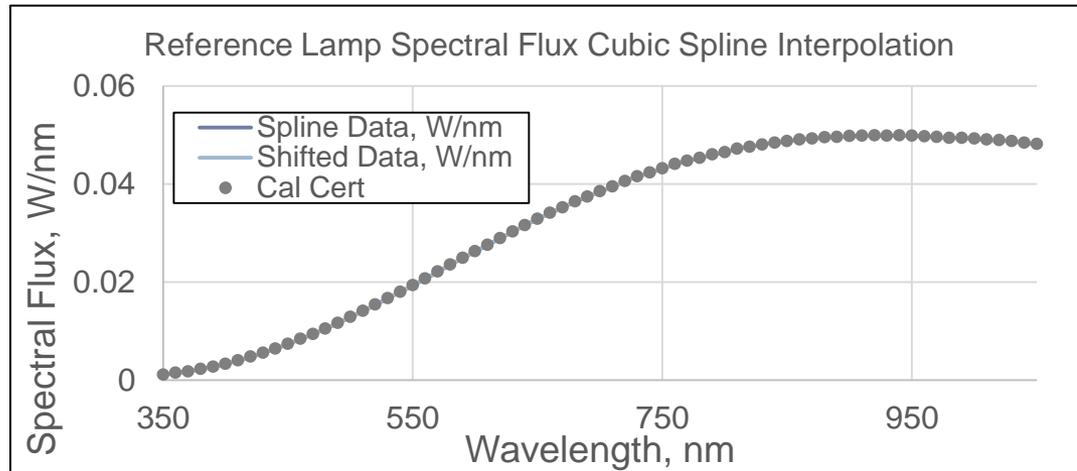
1. Interpolate $\phi_{R,\lambda}(\lambda)$ and $u_{\phi_{ref}}$ data from cal. Cert.
2. Calculate $\phi_{R,\lambda}(\lambda + \Delta\lambda)$
3. Calculate the difference: $\Delta\phi_{R,\lambda} = \phi_{R,\lambda}(\lambda + \Delta\lambda) - \phi_{R,\lambda}(\lambda)$
4. Calculate the Relative Uncertainty: $u_{\phi_{wavelength}} = \frac{\Delta\phi_{R,\lambda}}{\phi_{R,\lambda}}$

Combined Uncertainty: RSS the values:

$$u_{\phi} = \sqrt{u_{\phi_{ref}}^2 + u_{\phi_{wavelength}}^2}$$

NOTE: See A. Jackson, Corm 2014, for more rigorous approach to include uncertainty between data points.

Uncertainty Contributions: Ref. Lamp Calibration Certificate, $\phi_{R,\lambda}(\lambda)$



Uncertainty Contributions: Spectral Shift – Exponential Fit Method

$$C_{\phi}(\lambda) = \frac{(y_R(\lambda) - y_{R_d}(\lambda))}{\phi_{R,\lambda}(\lambda)} \cdot \frac{L_e(\lambda, T_0)}{L_e(\lambda, T)} \cdot \frac{(y_{xE}(\lambda) - y_{xE_d}(\lambda))}{(y_{xR}(\lambda) - y_{xR_d}(\lambda))} \cdot A_R C_{fR}$$

Spectral shift for a tungsten halogen lamp has been related to the blackbody radiation curve as:

$$\frac{L_e(\lambda, T_0)}{L_e(\lambda, T)} = \frac{\exp\left[a \left(\frac{J}{J_{ref}}\right)^{n_{JR}}\right] - 1}{\exp[a] - 1} \quad a = \frac{hc/\lambda}{kT_0}$$

h = Planck's Constant 6.6260957 E-34 m²kg/s

c = Speed of Light, 2.99792458E+13 nm/s

k = Boltzman Constant, 1.3806488E-23 m²kg/s²-K

T_0 = Color Temp from Ref. Lamp Calibration Cert., K

J_R = Reference Current from Calibration Cert., A

J = Current during Sphere Calibration, A

n_{JR} = CCT Sensitivity Exponent, 0.648

Uncertainty Contributions: Spectral Shift – Exponential Fit Method

$$\frac{L_e(\lambda, T_0)}{L_e(\lambda, T)} = \frac{\exp \left[a \left(\frac{J}{J_{ref}} \right)^{n_{JR}} \right] - 1}{\exp[a] - 1} \quad a = \frac{hc/\lambda}{kT_0}$$

Variable	Source	Uncertainty	Source
$J = 2.680A$	Current During Sphere Calibration	0.00379A	Spec Sheet
$n_{JR} = 0.648$	Calculated by measuring flux at different currents. Ref. 1, 2	0.005	Calculated

Ref. 1, C. Miller, NIST – measured data

Ref. 2, A. Jackson, CORM 2014 - methodology

Uncertainty Contribution: Lamp Power Supply Current

▶ Check the Spec sheets:

▶ Labsphere LPS-150

- ▶ Current accuracy = $\pm 0.1\%$
- ▶ Temperature Coefficient: $\pm 0.1\% / ^\circ\text{C}$ at 25C

$$\text{▶ } u_i = \sqrt{(0.001 i_{lamp})^2 + ((0.001(T_{amb} - 25C) i_{lamp})^2} = 0.00379A$$

▶ Agilent E3634A

- ▶ Depends on manual/computer setting
- ▶ Depends on range used
- ▶ Current Accuracy Good for 25C $\pm 5^\circ\text{C}$
 - Programming: $\pm(0.2\% + 10 \text{ mA})$
 - Readback: $\pm(0.15\% + 4\text{mA})$

$$\text{▶ } u_i = 0.0015 i_{lamp} + 0.004A = 0.00404A$$

For 2.680A @ 22C

Uncertainty Contributions:

Spectral Shift – Blackbody Method

- ▶ From theory and measurements, a 1% change in lamp current produces a 0.8% change in blackbody temperature and a 2.9% change in total power.
- ▶ Use Planck Blackbody function to calculate spectral flux at T.
- ▶ Total flux is given by Stefan-Boltzmann Equation.

$$\text{Planck's equation : } \phi_{\lambda}(T) = \frac{2\pi hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1} \left[\frac{W}{\mu m} \right]$$

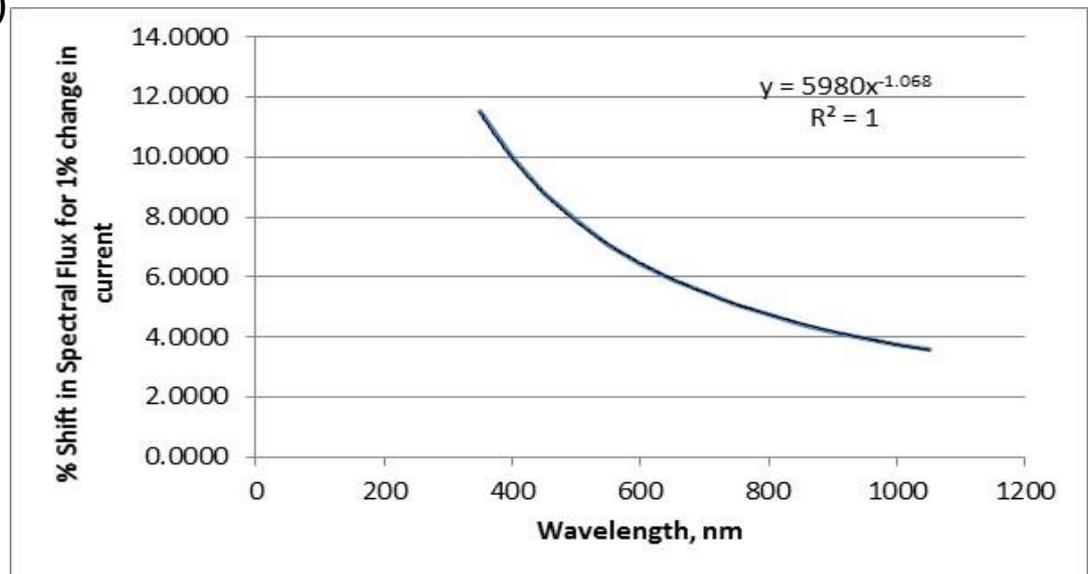
$$\text{Stefan-Boltzmann: } \phi(T) = \sigma T^4 [W_{lamp}]$$

$$\text{Spectral Flux per 1 W of Lamp Power at T: } F_{\lambda}(T) = \frac{2\pi hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1} \left(\frac{1}{\sigma T^4} \right) \left[\frac{W}{\mu m \cdot W_{lamp}} \right]$$

Uncertainty Contribution: Spectral Shift due to Lamp Current

► Calculate the Sensitivity of Spectral Flux due to spectral shift from lamp current uncertainty:

1. Calculate $F_{\lambda}(T)$ at CCT of Cal lamp, T_{cal} .
2. Calculate $F_{\lambda}(T)$ at $1.008 \times T_{cal}$.
3. Change in Spectral Flux for a 1% change in current due to spectral shift:
4. $\Delta\Phi(\lambda) = F_{\lambda}(T) - F_{\lambda}(T + 0.008T)$
5. Calculate the change in flux due to the uncertainty in the current:
6. $u_{\phi}(\lambda) = \left(\frac{u_i}{i_{cal}} \times 100\right) \Delta\Phi(\lambda)$



Uncertainty Contribution: Spectral Flux due to Cal Lamp Current

- ▶ The change in total power is 2.9% for a 1% change in current:

1. Calculate additional power due to uncertainty in current:

2. $u_{\phi}(\lambda) = \left(\frac{u_i}{i_{cal}} \times 100\right) 0.029\phi$

3. Total change in flux is the sum of each contribution.

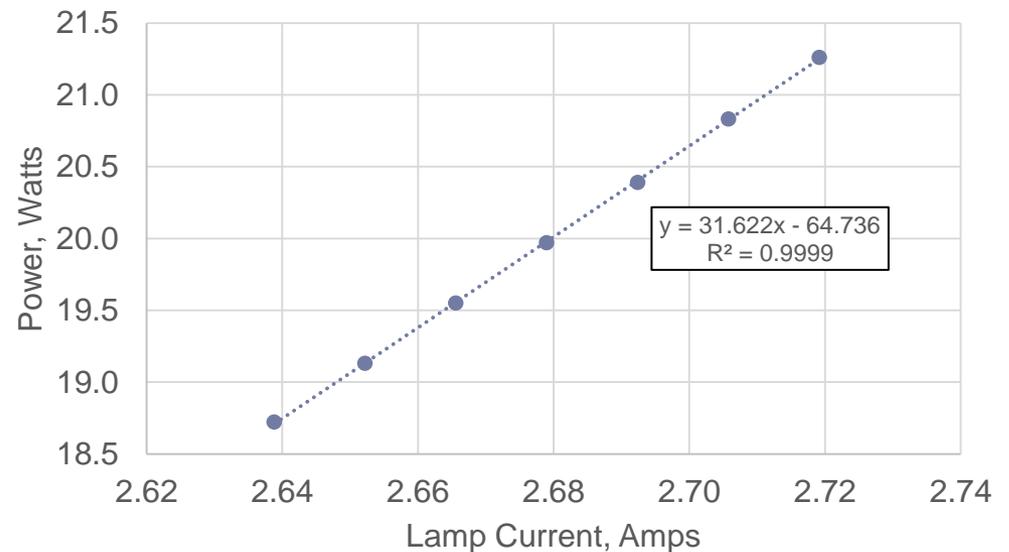
4. $u_{\phi}(\lambda) = \left(\frac{u_i}{i_{cal}} \times 100\right) \Delta\Phi(\lambda) + \left(\frac{u_i}{i_{cal}} \times 100\right) 0.029\phi$

Uncertainty Contribution: Total Flux due to Cal Lamp Current

- ▶ Estimate the change in flux due to lamp current by measurement.

1. Step lamp current between Nominal $\pm 3\%$
2. Measure Spectral Flux at Each Current Setting
3. Plot variation in total flux
4. $\frac{\partial \phi}{\partial i} = 31.622 \text{ W/Amp}$
5. Uncertainty in flux due to lamp current:

$$u_{\phi_i} = \left(\frac{\partial \phi}{\partial i} u_i \right)$$

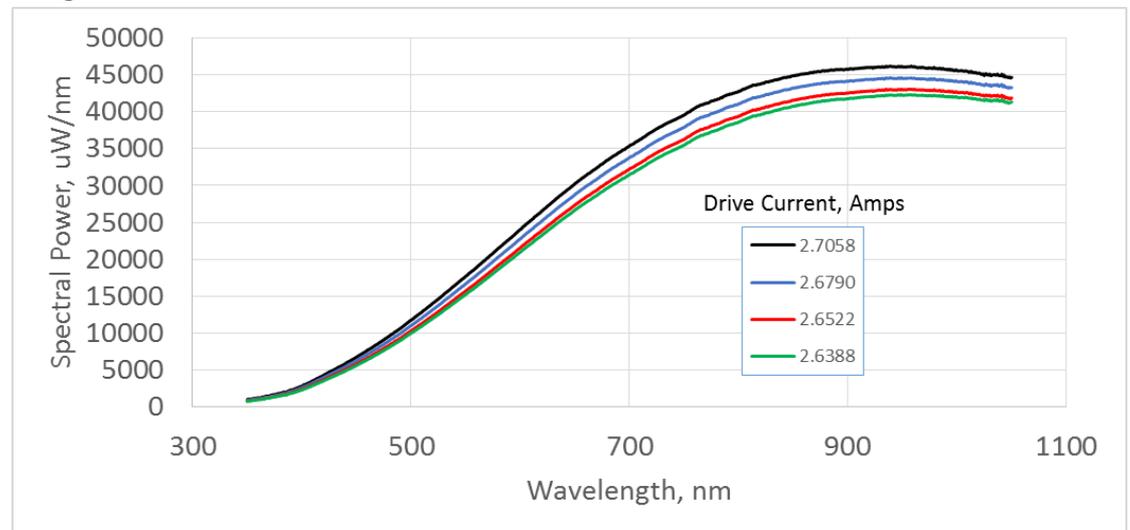


Uncertainty Contribution: Spectral Flux due to Cal Lamp Current

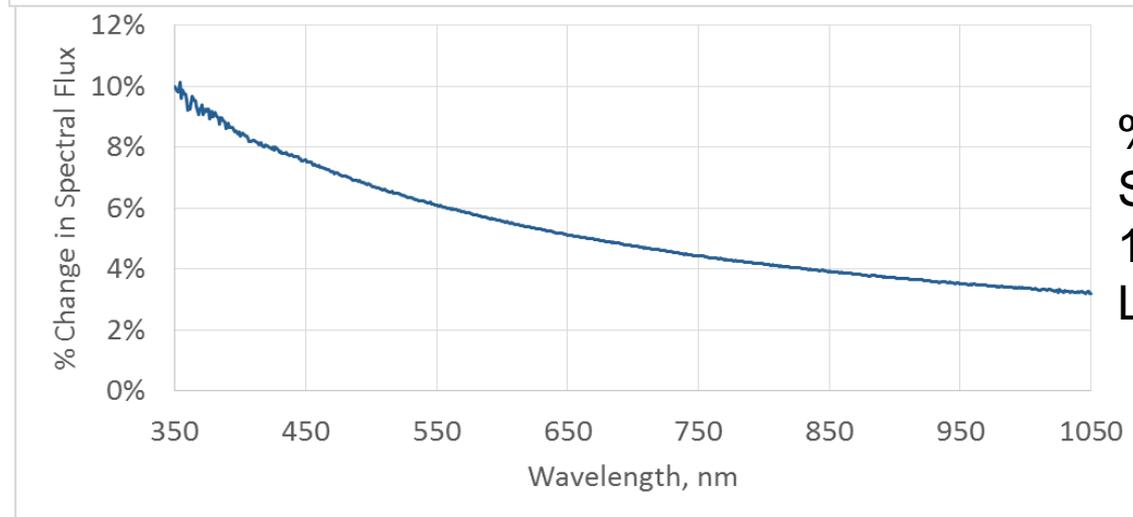
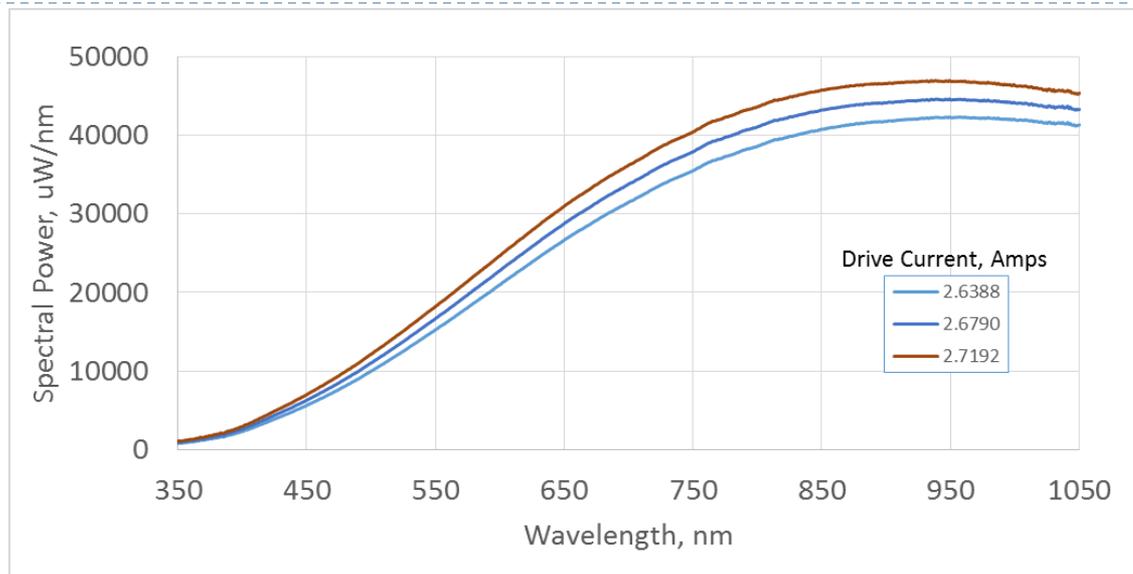
- ▶ Estimate the change in flux due to lamp current by measurement.

1. Step lamp currents for Nominal $\pm 3\%$
2. Measure Spectral Flux at Each Current Setting
3. Plot variation
4. Calculate $\frac{\partial \phi}{\partial i}$ at each wavelength
5. Uncertainty in flux due to lamp current:

$$u_{\phi_i}(\lambda) = \left(\frac{\partial \phi(\lambda)}{\partial i} u_i \right)$$



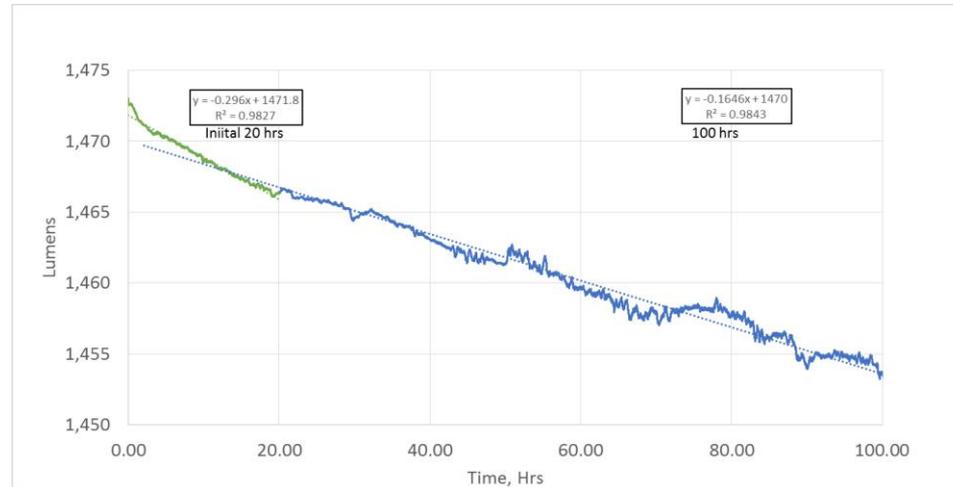
Uncertainty Contribution: Spectral Flux due to Cal Lamp Current



% change in Spectral Flux for 1% change in Lamp Current.

Uncertainty Contribution: Lamp Aging

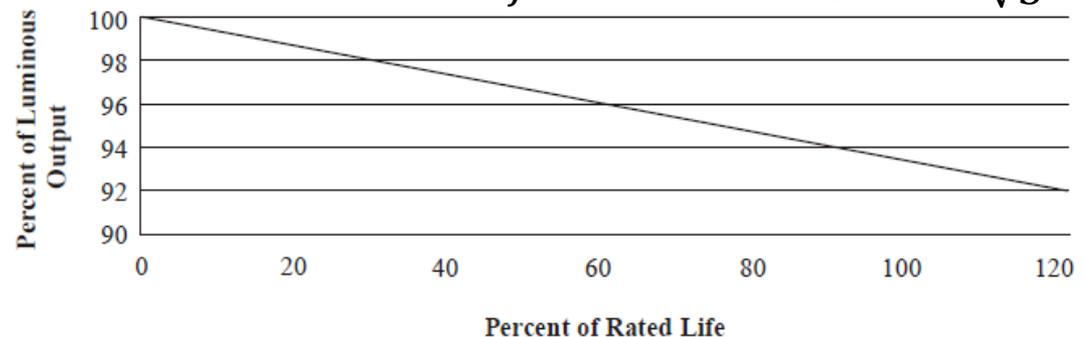
▶ Measure it:



▶ Model it:

- ▶ At 120% of lamp life, 8% degradation in flux.

$$u_{Lifetime}(\%) = \frac{Hrs\ Used}{Lifetime \times 120\%} \cdot 8\% \cdot \frac{1}{\sqrt{3}}$$

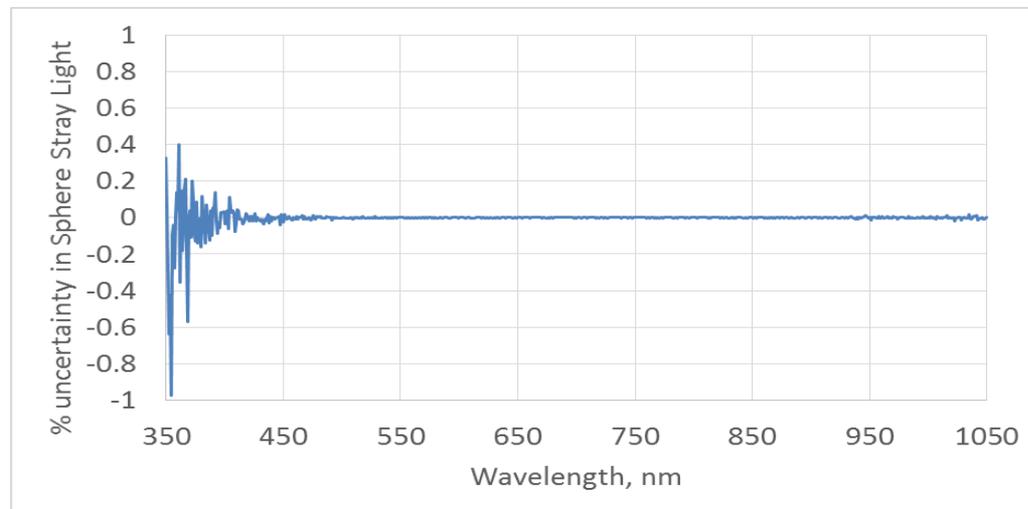


Stray Light into Sphere

▶ Measure it

- ▶ 10 scans with no system lamps on.
- ▶ Scans are relative to the signal from the cal lamp
- ▶ Normalize with integration time
- ▶ Divide by $\sqrt{3}$ for rectangular distribution

- ▶
$$u_{stray\ light}(\lambda) = \frac{\overline{y_{dark}}}{\overline{y_{ref}}} \cdot \frac{t_{dark}}{t_{ref}} \cdot \frac{1}{\sqrt{3}}$$



Near Field Absorption

- ▶ At close distances the surface may not produce a Lambertian reflectance.
- ▶ The amount of light absorbed by surfaces that do not have reflectance equivalent to the overall sphere.
- ▶ Estimate the amount of light that strikes the holder.
 - ▶ Assume 5% of the light interacts with near-field, f_{nf}
 - ▶ Assume 10% of this light is absorbed, α_{nf} , reducing the measurement:
$$u_{nf}(\lambda) = f_{nf} \times \alpha_{nf} \times 100\% = 0.5\%$$



Sphere Non-uniformity

- ▶ Measure it:

- ▶ Rotate cal lamp in 5° increments about center post.
- ▶ Measure Radiant Flux and normalize by mean value.
- ▶ From previous data, we measured Coeff. of Variation in Radiant Flux for 10 stationary scans = 0.09%.



- ▶ Variations > 0.9% influence of uniformity.

- ▶ Use a Constant value:

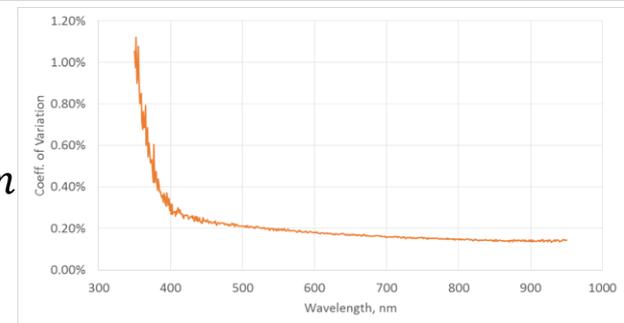
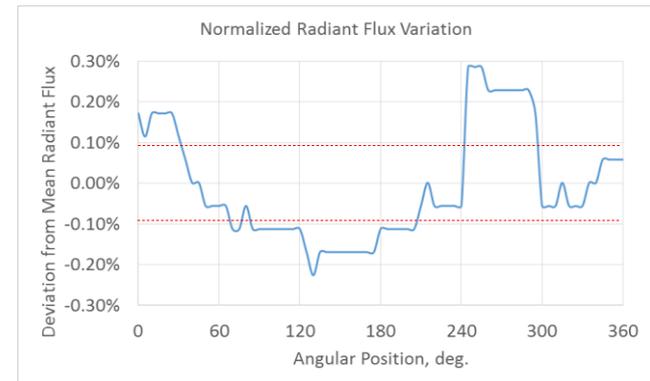
$$u_{nu} = 0.2\%$$

- ▶ Function of Wavelength:

- ▶ Std. Dev at each wavelength
across the rotation angles.

$$u_{nu}(\lambda) \approx -1.48E - 4\lambda + 6.11E - 2, 350 < \lambda < 400 \text{ nm}$$

$$u_{nu}(\lambda) \approx 0.2\%, 400 < \lambda < 1050 \text{ nm}$$



Spectrometer Stray Light

- ▶ Errors due to light reaching unintended pixels in spectrometer.
- ▶ Greatest impact at low wavelengths when measuring blue LEDs after calibration with QTH lamp.
- ▶ Measurements indicate 1% impact in radiant flux at 350-400 for measuring blue LEDs from QTH calibration.
- ▶ Minimal effects for $\lambda > 450\text{nm}$ for all lamps.
- ▶ $u_{sl}(\lambda) = 1\%$, 350 – 450 nm for Blue LED
 $u_{sl}(\lambda) = 0$ for $\lambda > 450$ nm.

Spectrometer Nonlinearity

- ▶ Effect of measuring a device whose output is orders of magnitude different than the calibration lamp.
- ▶ Define the photopic ratio:

$$P.R. = \text{Photopic ratio} = \frac{\varphi_{DUT}}{\varphi_{Cal lamp}}$$

- ▶ Base the uncertainty on the dynamic range of the spectrometer:

$$u_{nl} = 0.1 \frac{|\log(P.R.)|}{\log(60)} \quad \text{for CDS-1100/2100}$$

$$u_{nl} = 0.3 \frac{|\log(P.R.)|}{\log(60)} \quad \text{for CDS-600/610}$$

Summary: Inputs required

- ▶ Ref. Lamp Calibration Certificate
 - ▶ Color Temperature
 - ▶ Lamp Current
 - ▶ Radiant Flux
- ▶ Component information
 - ▶ Spectrometer Spec Sheet
 - ▶ Dynamic Range
 - ▶ Wavelength Accuracy
 - ▶ Stray Light information
 - ▶ Reference Lamp Hours Used
 - ▶ Power Supply Spec Sheets
 - ▶ Accuracy
 - ▶ Temperature Dependence
- ▶ Sphere
 - ▶ Uniformity
 - ▶ Near Field Absorption
- ▶ Measurements (multiple scans)
 - ▶ Integration times for each scan
 - ▶ Calibration Lamp Scans
 - ▶ Aux/Cal Scans
 - ▶ Aux/Empty Scans

Summary: Uncertainty Sources

- ▶ Where do I get my Estimates?
 - ▶ Existing Literature
 - ▶ Measure it
 - ▶ Spec Sheets
 - ▶ Derive it
- ▶ Things to Look out for
 - ▶ Does the literature data apply to my system?
 - ▶ What am I capturing in the measurements?
 - ▶ Does my measurement isolate the sensitivity of terms?
 - ▶ Is Spec sheet information sufficient?
 - ▶ Did I validate my derivations?
 - ▶ What are my assumptions

Summary: Sample Output

Uncertainty Budget at 550nm									
	Component Description	Symbol	Value	Unit	Standard Uncertainty, $u(x_i)$	Type	DOF, ν_i	Sensitivity, c_i	Contribution, $u_i(y)$
1	Spectral Flux Reference Standard	$\phi_{R,\lambda}(\lambda)$	0.019	W/nm	3.631E-04	B	∞	4.055E+03	1.472E+00
2	Reference Standard Scan	$y_R(\lambda)-y_{Rd}(\lambda)$	1.419	counts/nm/ms	4.845E-04	A	2	5.516E+01	2.673E-02
3	Empty Sphere Scan with Aux	$y_E(\lambda)-y_{Ed}(\lambda)$	8.491	counts/nm/ms	5.419E-03	A	∞	2.701E-03	4.996E-02
4	Aux scan with Reference Standard	$y_{xRd}(\lambda)-y_{xRd}(\lambda)$	7.973	counts/nm/ms	2.325E-03	A	∞	9.817E+00	2.282E-02
5	Reference Lamp Current	J	2.679	A	2.701E-03	B	2	1.891E+01	5.108E-02
6	Spectral Shift exponent	n_{JR}	0.648	-	2.887E-03	B	2	1.367E-01	3.945E-04
7	Lamp aging factor	A_R	0.00003	-	2.983E-05	B	∞	7.827E+01	2.335E-03
8	Additional Uncertainty Contributions	C_f	0.002	counts/nm/ms/W	1.803E-03	B	∞	7.827E+01	1.411E-01

Wavelength:	550
$C_\phi(\lambda)$	78.27
Expanded Uncertainty	2.90
Uncertainty, %	3.7%

