CCPR K3 Key Comparison of Luminous Intensity

Arnold Gaertner
NRC Metrology

CORM 2019 Annual Technical Conference
2019-October-28
Intensity

Radiant Intensity

\[ I_e(\lambda) = \frac{\Phi_e(\lambda)}{\Omega} = \frac{\text{radiant flux}}{\text{solid angle}} \]

unit = watt per steradian
Luminous Intensity

\[ I_v = \frac{\Phi_v}{\Omega} = \frac{\text{luminous flux}}{\text{solid angle}} \]

unit = candela = lumen per steradian

\[ \Phi_v = K_{cd} \int_{360 \, \text{nm}}^{830 \, \text{nm}} V(\lambda) \cdot \Phi_e (\lambda) \cdot d\lambda \]

unit = lumen

\[ K_{cd} = 683 \frac{\text{lumen}}{\text{watt}} \]
Intensity

Luminous Intensity, SI unit candela

\[ I_v = \frac{\Phi_v}{\Omega} \quad \text{unit} = \text{lumen per steradian} = \text{candela} \]

CGPM definition:

The candela, symbol cd, is the SI unit of luminous intensity in a given direction. It is defined by taking the fixed numerical value of the luminous efficacy of monochromatic radiation of frequency \(540 \times 10^{12}\) Hz, \(K_{\text{cd}}\), to be 683 when expressed in the unit \(\text{lm} W^{-1}\), which is equal to \(\text{cd sr} W^{-1}\), or \(\text{cd sr} \text{ kg}^{-1} \text{ m}^{-2} \text{ s}^3\), where the kilogram, metre and second are defined in terms of \(h, c\) and \(\Delta\nu_{\text{Cs}}\).

https://www.bipm.org/en/measurement-units/base-units.html
CCPR KEY COMPARISON CCPR-K3.2014
Comparison Organisation

• Selection of participants, artifacts and protocol

Comparison Procedures

• Comparison measurements and measurement verification
• Data analysis and comparison of participant SI candela realisations
• Write the report
CCPR Key Comparison CCPR-K3.2014

Comparison Organisation
- Selection of NRC as pilot
- Selection of participants (12 max)
- Task Group
  - Selection of artifact
    - Lamp vs photometer: standards-quality incandescent lamps
    - Type of lamp: Incandescent (Osram Wi41/G and NPL/Polaron heavy current)
    - Type of comparison (star type: participant – pilot – participant)
      - Standard lamps are fragile and expensive
  - Draft the technical protocol (artifact transportation, measurement reporting, uncertainties, etc.)
- Register the comparison: CCPR-K3.2014

<table>
<thead>
<tr>
<th>RMO Group</th>
<th>RMO Group Members</th>
<th>Maximum Number of Participants</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group 1</td>
<td>EURAMET+COOMET</td>
<td>6</td>
</tr>
<tr>
<td>Group 2</td>
<td>APMP+AFRIMETS</td>
<td>4</td>
</tr>
<tr>
<td>Group 3</td>
<td>SIM</td>
<td>2</td>
</tr>
</tbody>
</table>
## CCPR Key Comparison CCPR-K3.2014

### Comparison Organisation

- **Selection of participants**

<table>
<thead>
<tr>
<th>NMI</th>
<th>Country</th>
</tr>
</thead>
<tbody>
<tr>
<td>NMISA</td>
<td>South Africa</td>
</tr>
<tr>
<td>NIM</td>
<td>China</td>
</tr>
<tr>
<td>NMIA</td>
<td>Australia</td>
</tr>
<tr>
<td>NMIJ</td>
<td>Japan</td>
</tr>
<tr>
<td>IO-CSIC</td>
<td>Spain</td>
</tr>
<tr>
<td>LNE-CNAM</td>
<td>France</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>NMI</th>
<th>Country</th>
</tr>
</thead>
<tbody>
<tr>
<td>METAS</td>
<td>Switzerland</td>
</tr>
<tr>
<td>NPL</td>
<td>UK</td>
</tr>
<tr>
<td>PTB</td>
<td>Germany</td>
</tr>
<tr>
<td>VNIIOFI</td>
<td>Russia</td>
</tr>
<tr>
<td>NIST</td>
<td>USA</td>
</tr>
<tr>
<td>NRC</td>
<td>Canada</td>
</tr>
</tbody>
</table>
Comparison Artifact

- Type of lamp: Incandescent (Osram Wi41/G and NPL/Polaron heavy current)
Comparison Procedures • Measurements

• Comparison measurements
  • Each participant supplied their own calibrated (~6) lamps (ship or hand-carry)
  • NRC received and measured ~70 comparison lamps
  • Each participant re-measured their lamps

• Measurement verification and artifact certification
  • Each participant compares before and after shipment measurements
  • NRC provides relative data for all the artifacts of each participant
  • Removal of unstable artifacts => final comparison artifacts
Comparison Procedures • Data Analysis
Comparison Procedures • Analysis

• Data analysis
  • Determine final NRC measurement value for each artifact
  • Determine final NRC measurement value for each participant

• Comparison of participant SI candela realisations
  • KCRV (Key Comparison Reference Value)
    • Weighted mean with cut-off
  • Identification of ‘outliers’: deviation from KCRV greater than 6 times their uncertainty
  • Consistency check: Chi-square($\alpha = 0.05$) test, $\chi^2_{0.05}(\nu = 11) = 19.7$

• All this requires an uncertainty analysis (NRC and Participant measurements)
Comparison Procedures ● Measurements at pilot (NRC)

- Comparison of all artifacts under identical measurement configuration
Comparison Procedures • Measurements at pilot (NRC)

- Comparison of all artifacts under identical measurement configuration

\[
R_{i,j,m} = \frac{I_{V(i,j)}}{V_{i,j,m}} \left( \frac{\text{cd}}{\text{volt}} \right)
\]

\[
I_{V(i,j)} \text{ (cd)}
\]

\[
V_{i,j,m} \text{ (volt)}
\]
Comparison Procedures • Measurements at pilot (NRC)

- Comparison of all artifacts under identical measurement configuration
- $d \sim 3.2 \, m$
- 3 photometers
- $\geq 2$ measurements/lamp
- $\sim 250$ measurements
- $\sim 2$ months
Comparison Procedures • Measurements at pilot (NRC)

- **How accurate is the comparison?**

- **Sources of Uncertainty •** \( u(V_{i,j,m}) \) • (~15!)
  - NRC Optical Coordinate System (2)
  - NRC Photometer (5)
  - Participant Lamps
    - Electrical (4)
    - Optical (3)
    - Photometric (1)

**Consider:**

- In 3D space there are 6 variables: 3 spatial and 3 angular
- Lamp output: % change \( \approx 7 \) times % change in lamp current
  - Am I operating the lamp electricals to the same standards as the participant?
- How/with what do I ensure stability over 2 months of measurements?
CCPR Key Comparison CCPR-K3.2014

Comparison Procedures • Measurements at pilot (NRC)

• Sources of Uncertainty • $u(V_{i,j,m})$
  • NRC Optical Coordinate System (2)
    • Starting line is X-axis (laser beam)
    • Alignment of Y-axis to X-axis (laser)
    • Alignment of Z-axis to XY axes
CCPR Key Comparison CCPR-K3.2014

Comparison Procedures • Measurements at pilot (NRC)

• Sources of Uncertainty • \( u(V_{i,j,m}) \)
  - NRC Optical Coordinate System (2)
  - NRC Photometer (5)
  - Spectral Mismatch Error
    \[
    F^* = \frac{\int_{360\text{ nm}}^{830\text{ nm}} P_e^T(\lambda) \cdot V(\lambda) \cdot d\lambda}{\int_{all\ wavelengths} P_e^T(\lambda) \cdot R(\lambda) \cdot d\lambda} \cdot \frac{\int_{all\ wavelengths} P_e^S(\lambda) \cdot R(\lambda) \cdot d\lambda}{\int_{360\text{ nm}}^{830\text{ nm}} P_e^S(\lambda) \cdot V(\lambda) \cdot d\lambda}
    \]
  - Responsivity Drift (what is constant over the 2 months of measurements?)
  - Signal Noise (fluctuations)
  - Alignment to optical axis (Y-Z centre)
  - Alignment to optical axis (Y-Z angular)
Comparison Procedures ● Measurements at pilot (NRC)

• Sources of Uncertainty ● $u(V_{i,j,m})$
  - NRC Optical Coordinate System (2)
  - NRC Photometer (5)
  - Participant Lamps
    - Electrical (4)
      - Standard Resistor calibration (lamp current measurement)
      - DVM voltage calibration (lamp current measurement)
      - Lamp current setting
      - Lamp current fluctuations
        - % change in lamp output is approximately 7 times % change in lamp current
    - Optical (3)
    - Photometric (1)
CCPR Key Comparison CCPR-K3.2014

Comparison Procedures • Measurements at pilot (NRC)

• Sources of Uncertainty • $u(V_{i,j,m})$
  • NRC Optical Coordinate System (2)
  • NRC Photometer (5)
  • Participant Lamps
    • Electrical (4)
    • Optical (3)
      • Vertical filament plane (parallel to Z-axis, rotation about Y-axis)
      • Vertical filament plane (parallel to Y-axis, rotation about Z-axis)
      • Lamp to photometer distance (photometer signal $\propto \frac{1}{d^2}$)
  • Photometric (1)
    • Lamp output fluctuations
Comparison Procedures • Measurements at pilot (NRC)

• Sources of Uncertainty • Summary • $u(V_{i,j,m})$

- 4 predominant sources of uncertainty:

<table>
<thead>
<tr>
<th>Source of Uncertainty</th>
<th>Type</th>
<th>Relative Standard Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>NRC Photometer</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Spectral Mismatch Error</td>
<td>B</td>
<td>0.01%</td>
</tr>
<tr>
<td>Responsivity Drift</td>
<td>A</td>
<td>0.05%</td>
</tr>
<tr>
<td>Participant Lamps (optical)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Vertical Filament Plane</td>
<td>A</td>
<td>0.01%</td>
</tr>
<tr>
<td>Lamp-to-Photometer distance</td>
<td>A</td>
<td>0.03%</td>
</tr>
</tbody>
</table>
Comparison Procedures • Data Analysis

- Sources of Uncertainty
  - 3 sources:
    - Participant LI values
    - NRC comparison measurements
    - Artifact repeatability at NRC
  - Kinds of uncertainties:
    - Type A
    - Type B
    - Uncorrelated
    - Correlated

\[ R_{i,j,m} = \frac{I_v(i,j)}{V_{i,j,m}} \left( \frac{\text{cd}}{\text{volt}} \right) \]
Comparison Procedures • Data Analysis

• Sources of Uncertainty • combination of uncertainties*
  
  • Kinds of Uncertainties:
  • Type A
  • Type B
  • Uncorrelated (uc)
  • Correlated (c)

\[
Q = f(x_i)
\]

\[
u_{uc}^2(Q) = \sum_{i=1}^{n} \left( \frac{\partial f}{\partial x_i} \right)^2 \cdot u_{uc}^2(x_i)
\]

\[
u_c^2(Q) = \left[ \sum_{i=1}^{n} \left( \frac{\partial f}{\partial x_i} \right) \cdot u_c(x_i) \right]^2
\]

\[
u_{total}^2(Q) = u_{uc}^2(Q) + u_c^2(Q)
\]

• *GUM, Guides to the expression of uncertainty in measurement, JCGM 100:2008, etc. www.bipm.org
Comparison Procedures • Data Analysis

- Sources of Uncertainty • combination of uncertainties • weighted mean

- Weights \( W_i = \frac{1}{u_i^2} \)
- Normalised \( w_i = \frac{w_i}{\sum w_i} \)

\[
Q = \sum_{i=1}^{n} w_i \cdot x_i
\]

\[
\frac{\partial Q}{\partial x_i} = w_i
\]
## CCPR Key Comparison CCPR-K3.2014

### Comparison Procedures • Data Analysis

- **Sources of Uncertainty** • combination of uncertainties • weighted mean
  - Type A
  - Type B
  - Uncorrelated
  - Correlated

<table>
<thead>
<tr>
<th>Measured Quantity</th>
<th>Uncertainty</th>
<th>Combined</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>$V_1$</strong></td>
<td>$u_A(V_1)$</td>
<td>$u_B(V_1)$</td>
</tr>
<tr>
<td><strong>$V_2$</strong></td>
<td>$u_A(V_2)$</td>
<td>$u_B(V_2)$</td>
</tr>
<tr>
<td><strong>$\ldots$</strong></td>
<td>$\ldots$</td>
<td>$\ldots$</td>
</tr>
<tr>
<td><strong>$V_n$</strong></td>
<td>$u_A(V_n)$</td>
<td>$u_B(V_n)$</td>
</tr>
</tbody>
</table>

| $f(V_i)$         | $u_A^2(f)$  | $u_B^2(f)$ | $\sqrt{u_A^2(f) + u_B^2(f)}$ |
|-------------------|-------------|-------------|
| $f(V_i) = \sum_{i=1}^{n} w_i V_i$ | $u_A^2(f) = \sum_{i=1}^{n} w_i^2 u_A^2(V_i)$ | $u_B^2(f) = \left[\sum_{i=1}^{n} w_i u_B(V_i)\right]^2$ | $\sqrt{u_A^2(f) + u_B^2(f)}$ |

Weighted mean  Uncorrelated  Correlated  Combined
Comparison Procedures • Analysis

• Data analysis

  • Determine final NRC measurement value for each artifact: \( R_{i,j} = \langle R_{i,j,m} \rangle_m \), \(~12\times6=72\) values
    
    • \( u(R_{i,j}) \) is a combination of NRC measurements \( (u_A \text{ and } u_B) \), Participant \( (u_A \text{ and } u_B) \) and lamp \( u_A \)

  • Determine final NRC measurement value for each participant: \( R_i = \langle R_{i,j} \rangle_j \), = 12 values
    
    • \( u(R_i) \) is a combination of the \( (u_A \text{ and } u_B) \) components of \( u(R_{i,j}) \)

• Comparison of participant SI candela realisations

  • KCRV (Key Comparison Reference Value)
    
    • Weighted mean with cut-off

  • Identification of ‘outliers’: deviation from KCRV greater than 6 times their uncertainty

  • Consistency check: Chi-square\( (\alpha = 0.05) \) test, \( \chi^2_{0.05}(v = 11) = 19.7 \)

\[ R_{i,j,m} = \frac{I_v(i,j)}{V_{i,j,m}} \left( \frac{\text{cd}}{\text{volt}} \right) \]
Comparison Procedures • Analysis

• Comparison of participant SI candela realisations
  • KCRV (Key Comparison Reference Value)
    • Weighted mean with cut-off

\[ u_{\text{cut-off}} = \text{average}(u_7 \text{ to } u_{12}) \]
Comparison Procedures • Analysis

• Comparison of participant SI candela realisations
  • KCRV (Key Comparison Reference Value)
    • Weighted mean with cut-off

\[ u_{adj}(R_i) = u_{adj}(NMI) + u_{transfer}(R_i) \]

weights \( W_{i,adj} = \frac{1}{u_{adj}(R_i)} \)

normalised \( w_{i,adj} = \frac{W_{i,adj}}{\sum W_{i,adj}} \)

Participant Luminous Intensity uncertainty
Relative standard values
(ordered highest to lowest)

<table>
<thead>
<tr>
<th>Participant</th>
<th>unadjusted</th>
<th>adjusted ( u_{adj}(NMI) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( u_1 )</td>
<td>( u_1 )</td>
<td></td>
</tr>
<tr>
<td>( u_2 )</td>
<td>( u_2 )</td>
<td></td>
</tr>
<tr>
<td>....</td>
<td>....</td>
<td></td>
</tr>
<tr>
<td>( u_6 )</td>
<td>( u_6 )</td>
<td></td>
</tr>
<tr>
<td>( u_7 )</td>
<td>( u_7 )</td>
<td></td>
</tr>
<tr>
<td>....</td>
<td>....</td>
<td></td>
</tr>
<tr>
<td>( u_{cut-off} )</td>
<td>( u_{cut-off} )</td>
<td>( u_{cut-off} )</td>
</tr>
<tr>
<td>( u_{11} )</td>
<td>( u_{cut-off} )</td>
<td>( u_{cut-off} )</td>
</tr>
<tr>
<td>( u_{12} )</td>
<td>( u_{cut-off} )</td>
<td>( u_{cut-off} )</td>
</tr>
</tbody>
</table>
CCPR Key Comparison CCPR-K3.2014

Comparison Procedures • Analysis

• Comparison of participant SI candela realisations
  • KCRV (Key Comparison Reference Value)
    • Weighted mean with cut-off

\[ R_{KCRV} = \sum_{i=1}^{n} w_{i,adj} \cdot R_i \left( \frac{cd}{volt} \right) \]

\[ u^2(R_{KCRV}) = \sum_{i=1}^{n} w_{i,adj}^2 \cdot u^2(R_i) \]

(uncorrelated)

Participant Luminous Intensity uncertainty
Relative standard values
(ordered highest to lowest)

<table>
<thead>
<tr>
<th>unadjusted</th>
<th>adjusted ( u_{adj}(NMI) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( u_1 )</td>
<td>( u_1 )</td>
</tr>
<tr>
<td>( u_2 )</td>
<td>( u_2 )</td>
</tr>
<tr>
<td>....</td>
<td>....</td>
</tr>
<tr>
<td>( u_6 )</td>
<td>( u_6 )</td>
</tr>
<tr>
<td>( u_7 )</td>
<td>( u_7 )</td>
</tr>
<tr>
<td>....</td>
<td>....</td>
</tr>
<tr>
<td>( u_j )</td>
<td>( u_j )</td>
</tr>
<tr>
<td>( u_k )</td>
<td>( u_{cutoff} )</td>
</tr>
<tr>
<td>....</td>
<td>( u_{cutoff} )</td>
</tr>
<tr>
<td>( u_{11} )</td>
<td>( u_{cutoff} )</td>
</tr>
<tr>
<td>( u_{12} )</td>
<td>( u_{cutoff} )</td>
</tr>
</tbody>
</table>

NATIONAL RESEARCH COUNCIL CANADA
Comparison Procedures ● Analysis

• Comparison of participant SI candela realisations
  • KCRV (Key Comparison Reference Value)
    • Weighted mean with cut-off
  • Identification of ‘outliers’: deviation from KCRV greater than 6 times their (k=1) uncertainty
  • Consistency check: Chi-square($\alpha = 0.05$) test, $\chi^2_{0.05}(v = 11) = 19.7$

$$\chi^2_{observed} = \sum_{i=1}^{n} \frac{(R_i - R_{KCRV})^2}{u^2_{adj}(R_i)}$$
Comparison Procedures • Analysis

- Comparison of participant SI candela realisations
  - KCRV (Key Comparison Reference Value)
    - Weighted mean with cut-off
  - Identification of ‘outliers’: deviation from KCRV greater than 6 times their uncertainty
  - Consistency check: Chi-square($\alpha = 0.05$) test, $\chi^2_{0.05}(v = 11) = 19.7$

IF $\chi^2_{observed} > \chi^2_{0.05}(v)$ (inconsistent!)

THEN add Mandel-Paule adjustment uncertainty $s$

$$u_{adj}^2(R_i) = u_{adj}^2(NMI) + u_{transfer}^2(R_i) + s^2$$

And REPEAT calculations with various $s$ until ‘consistent’
Comparison Procedures • Analysis

- Comparison of participant SI candela realisations
  - KCRV (Key Comparison Reference Value)
    - Weighted mean with cut-off
  - Identification of ‘outliers’: deviation from KCRV greater than 6 times their uncertainty
  - Consistency check: Chi-square($\alpha = 0.05$) test, $\chi^2_{0.05}(\nu = 11) = 19.7$
  - Calculate the Unilateral Degrees of Equivalence (DOE): $D_i$

\[
D_i = \frac{R_i - R_{KCRV}}{R_{KCRV}}
\]

\[
u_i^2 = u^2(R_i) + u^2(R_{KCRV}) - 2(w_i \cdot u^2(R_i))
\]

$R_i$ and $R_{KCRV}$ are correlated
Comparison Procedures • Data Analysis
Comparison Procedures ● Analysis

- Comparison of participant SI candela realisations
  - KCRV (Key Comparison Reference Value)
    - Weighted mean with cut-off
  - Identification of ‘outliers’: deviation from KCRV greater than 6 times their uncertainty
  - Consistency check: Chi-square ($\alpha = 0.05$) test, $\chi^2_{0.05}(\nu = 11) = 19.7$
  - Calculate the Unilateral Degrees of Equivalence (DOE)
  - Calculate the Bilateral Degrees of Equivalence

\[
D_{i,j} = \frac{R_i - R_j}{R_{KCRV}}
\]
\[
u_{i,j}^2 = u^2(R_i) + u^2(R_j)
\]

($R_i$ and $R_j$ uncorrelated)
Comparison Organisation

• Selection of participants, artifacts and protocol

Comparison Procedures

• Comparison measurements and measurement verification
• Data analysis and comparison of participant SI candela realisations
• Write the report
  • Draft A and any revisions, confidential to participants
  • Draft B to CCPR WG-KC for approval (and/or any revisions)
  • Approved Draft B to CCPR for approval
  • Final Report
CCPR KEY COMPARISON CCPR-K3.2014
ACKNOWLEDGEMENTS

W.S. Neil  R.J. Douglas
Éric Côté   J.C. Zwinkels
12 NMI participants
THANK YOU

Arnold Gaertner• Research Officer• arnold.gaertner@nrc-cnrc.gc.ca