Measuring the Luminous Effectiveness of Images

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OVERVIEW

To explain the "images" part of this presentation, I'll begin with a brief review of some the fundamental principles of photometry that are focussed on specifying light sources - be they lamps or surface reflections. Then I shall suggest another perspective: *incident photometry* - measuring light as it is received by the eye or some other sensing device. From that perspective, a new photometric measurement is proposed. It takes into account that images are defined by sensor arrays.

PHOTOMETRY BASICS

Did some early science course show you a diagram like Figure 1 below?

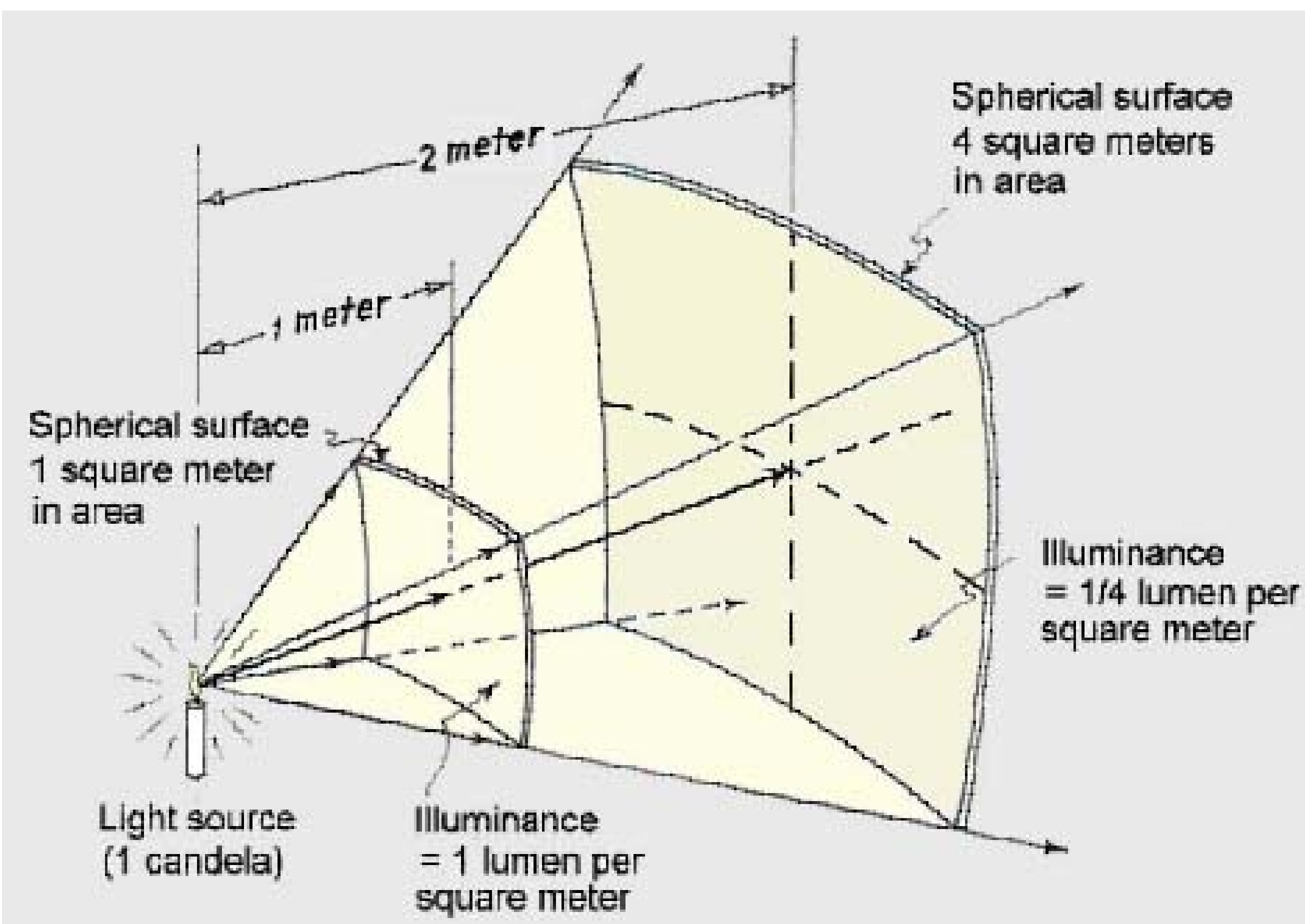


Figure 1. As a source radiates light in all directions, the light in any direction spreads. That spread is directly a function of the distance from the source squared. Therefore the amount of light on any unit area, such as the 1 meter square in the figure, diminishes with distance squared. If the source emits 1 candela, a 1 square meter surface at a distance of 1 meter receives 1 lumen of light. A 1 square meter surface at twice that distance receives only a quarter of the lumen.

Pity. It is a beautiful explanation of the **Inverse Square Law of Radiation.** But once seen, it is almost impossible to consider photometry as any thing other than a way of measuring the light emitted by some source.

To standardize the measurement of light in the above situation, we replace the candle in Figure 1by a black body at 2024^0 Kelvin with an aperture of 1/60th cm^2. Called a "**candela**", this source was chosen to closely approximate the light emitted by the previous standard - an actual whale oil candle. One meter in front of the candela is placed a 1 square meter photo-electric light detector connected to a meter that reads either the current, voltage or resistance of the detector. Whatever the meter reads corresponds to a radiant flux of 1 joule per second or 1 "**watt**". If the detector's spectral sensitivity matches that of the human eye, whatever the meter reads indicates a luminous flux of 1 "**lumen**".

If the detector has a linear (or correctable to linear) response with respect to the rate of energy it receives, it can be used to compare the luminous output of any point source of light.

Rather than carrying around detectors the size of a square meter, one can measure light relative to detector size by dividing a calibrated detector's lumen reading by the detector's area. The result describes the density of the luminous flux and is called "**illuminance**". Its units of measurement are "**lumen per meter^2**". Density of luminous flux is usually applied in other contexts and often referred to as "**lux**".

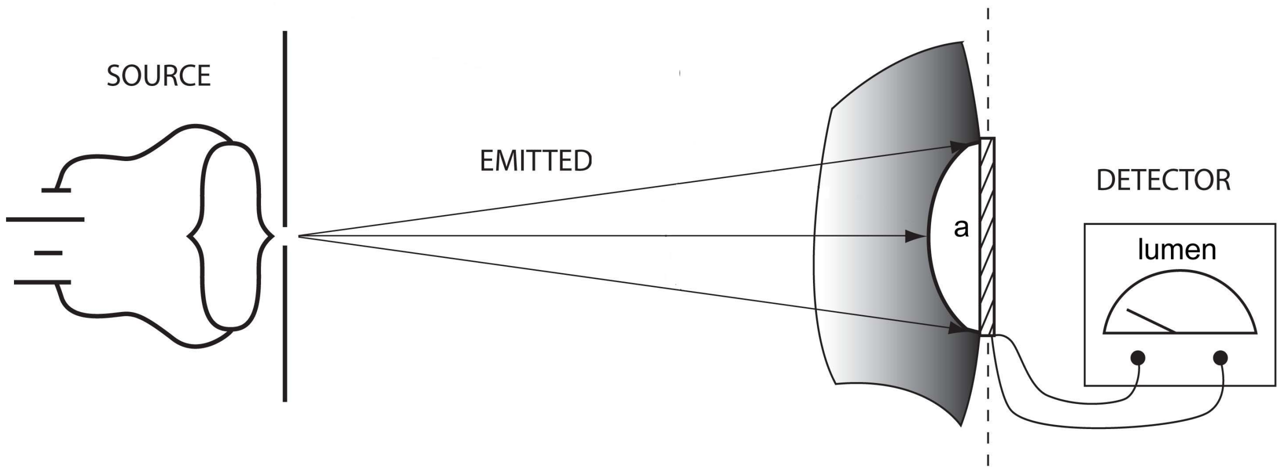


Figure 2. Once calibrated, any size detector can measure the density of luminous flux by dividing its lumen reading by the detector's area "a".

To specify point sources, both detector size and distance can be lumped into a single concept - direction.

It is easy to think of direction as a vector - a straight line pointed in some direction based on a 3-dimensional protractor. However, a vector is useless in dealing with energy because an infinitely thin line could not contain any photons. To encompass finite space, the basic physical unit of direction is a solid angle called a "**steradian**".

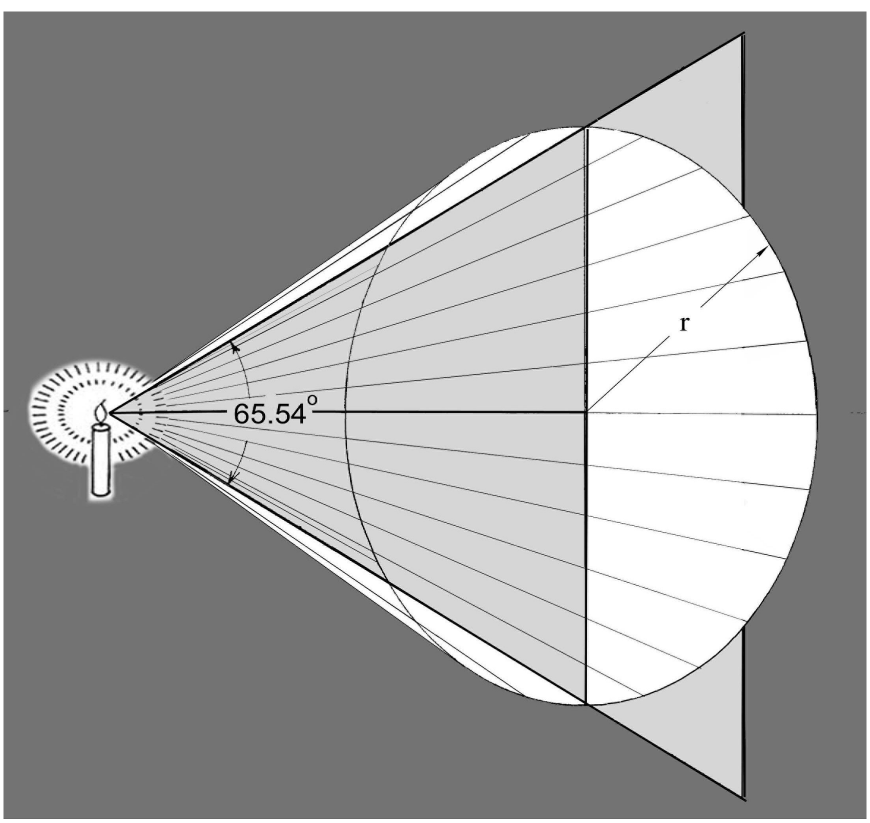


Figure 3. Unlike a direction, the steradian is a finite physical measure of direction that can convey energy. A simple geometric definition results in the steradian being rather large. Therefore, milli- and micro-steradians are often used, eg. the moon subtends 60 micro-steradians.

To keep it geometrically simple, the unit of direction is defined as the solid angle that a surface area of r^2 subtends with respect to a point at a distance “r”.

Unfortunately, this makes a **steradian** huge with respect to most matters optical.

The steradian is difficult to conceptualize. Its size may be better grasped by thinking of it as a 360^0 rotated plane angle of **65.5 degrees**.

Dividing a detector’s reading in lumen by the detector’s solid angle with respect to the source is a measurement called “**luminous intensity**”. Its unit of measurement, “lumen per steradian”, is generally combined into a single unit - “**candela**”. It specifies the luminous flux that a source emits in a certain direction.

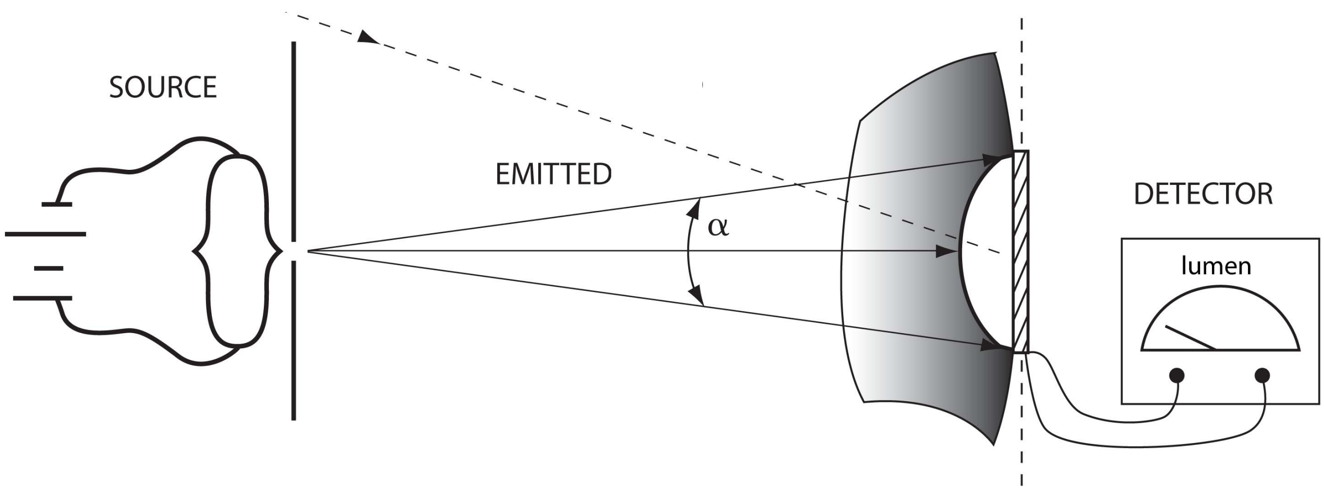


Figure 4. A detector of any size calibrated in lumen can be used to measure **luminous intensity** in candela at any distance from a point source by dividing its reading by the solid angle it subtends with respect to the source: candela = lumen / α .

Sensory experience can not be measured directly. A basic tenet of **psychophysics** is that a sensory experience can measured objectively in terms of the stimulus properties required to produce that experience. For example: “How much light is needed to see a **STOP** sign?” “How much of difference in wavelength is needed to see a difference in **color**?” Using various experimental designs, it is possible to define almost any sensory experience in terms of some objective physical measurement of the stimuli by using simple, nearly-objective responses such as “I see it.”, or “I see a difference.” Thus, vision researchers learn how to specify their stimuli using photometry.

Yet there is something wrong here. Thanks to the physicists and engineers who developed our light meters, we think about photometry the way they do - a means of measuring light sources. However, the only light that matters for vision is the light entering the eye. And when you think about it, incident light is also what actually drives the photo-detectors in our photometers. It is by inference that we relate the measurement to the light source itself.

To explain how images can measured, I ask you not think about photometry as a means of measuring light sources. Instead, I want to introduce you to:

INCIDENT LIGHT PHOTOMETRY

Thanks to the *Time Reversibility* of the laws of physics. The direction in which light travels makes no difference. On that basis, viewing can be interpreted as the reverse of radiating. (Plato was correct after all. It just did not come across in translation.)

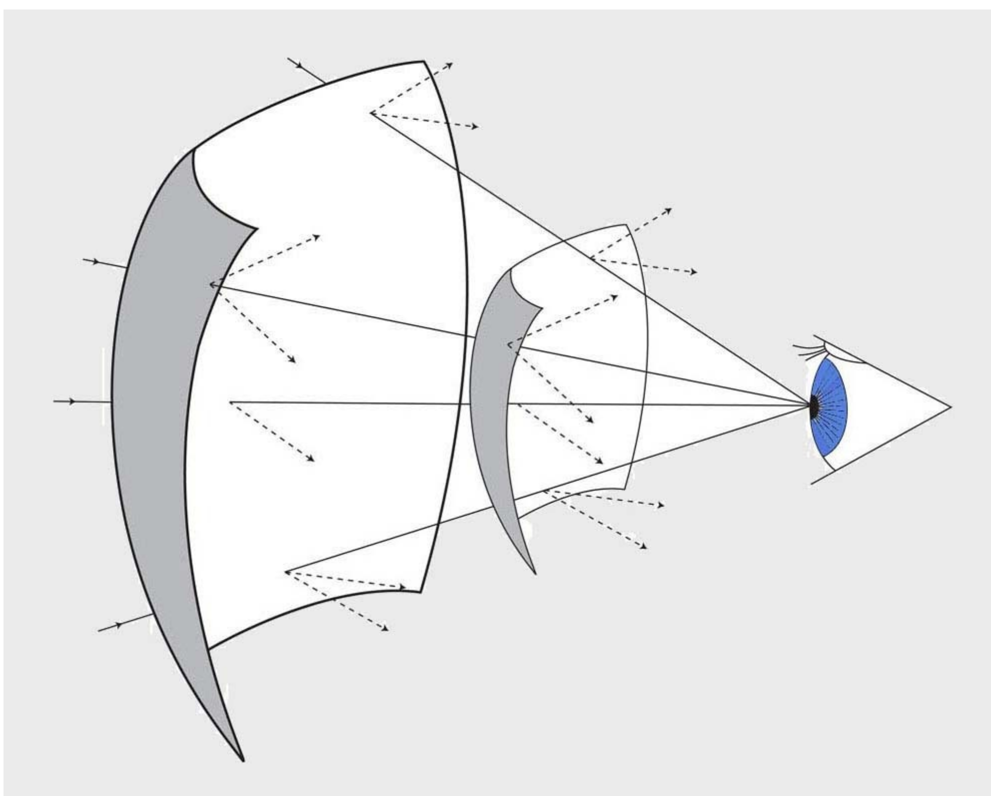


Figure 5. Viewing is the reverse of radiating.

Incident Luminous Intensity

All basic photometric measurements of light sources have a corresponding measurement for the amount of light received. For example, the amount of light emitted in a certain direction, luminous intensity, has a corresponding measurement - the amount of light received **from** a certain direction.

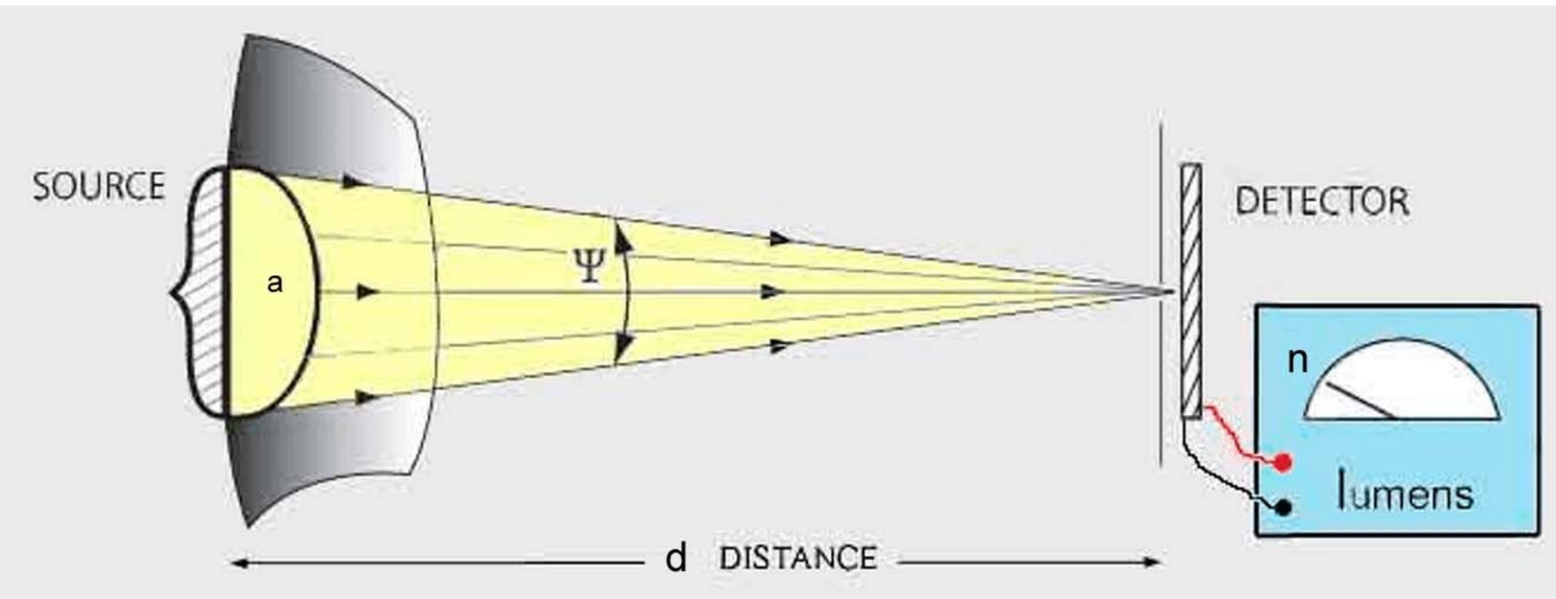


Figure 6. Measuring the amount of light received from a certain direction. If the source has an area equal to “a” meters^2 and is located “d” meters from the detector, its solid angle, Ψ, with respect to the detector equals a / d^2 steradians. Say the meter reads “n“ lumen. Dividing “n” by Ψ steradians produces a result that has the same physical units as the “candela” used for luminous intensity:

lumen/steradian

Measuring the light received from a certain direction uses the same distance between source and detector as measuring the light emitted by a point source. Both result in lumen per steradian or candela. Both refer to a concentration of luminous flux at a point. Therefore it seems warranted to refer to both as measures of luminous intensity. Yet since they differ in using either the area of the light or the area of the detector, confusion may avoided by referring to them as **incident** or **emitted** **luminous intensity** .

Incident Illuminance?

Illuminance is the only traditional photometric which refers to incident light. Its emitted counterpart, "**luminous exitance**, is the flux density of light leaving a surface such as a luminescent panel. The only difference is the direction of the direction of time. Since no spatial component is involved, calculation of either measurement is the same. The name is changed to protect the innocent:

illuminance or exitance = lumen / detector surface area

Incident Luminance

Point detectors are not a sensitive means of measuring incident light. For the same reason visual systems employing point receptors are rare. Unless you are studying certain animals like the *Nautilus*, you probably find luminous intensity only of pedagogical value.

Incident luminance measures the luminous flux incident on an extended detector from a particular direction. This is representative of most vision systems, which have an extended retina or electronic photoreceptor. This makes it the best classical photometric unit for describing light in terms that relate to the experience of brightness.

Figure 7 shows how incident luminance is measured. An extended detector receives light from an extended distal source. Say a detector responds to the incident luminous flux with a reading of "L" lumen per meter^2. That reading depends on the size of the source. However, each part of a uniform light source
looks the same brightness regardless of source size. This is solved by dividing "L" by the source's solid angle to obtain a measurement per solid angle of the source. The reading also depends on the area of the detector. This is avoided by dividing the result by detector area to obtain:

$$\textbf{incident luminance = L lumen / solid angle}_{\textit{source}}\textbf{ / area}_{\textbf{detector}}$$

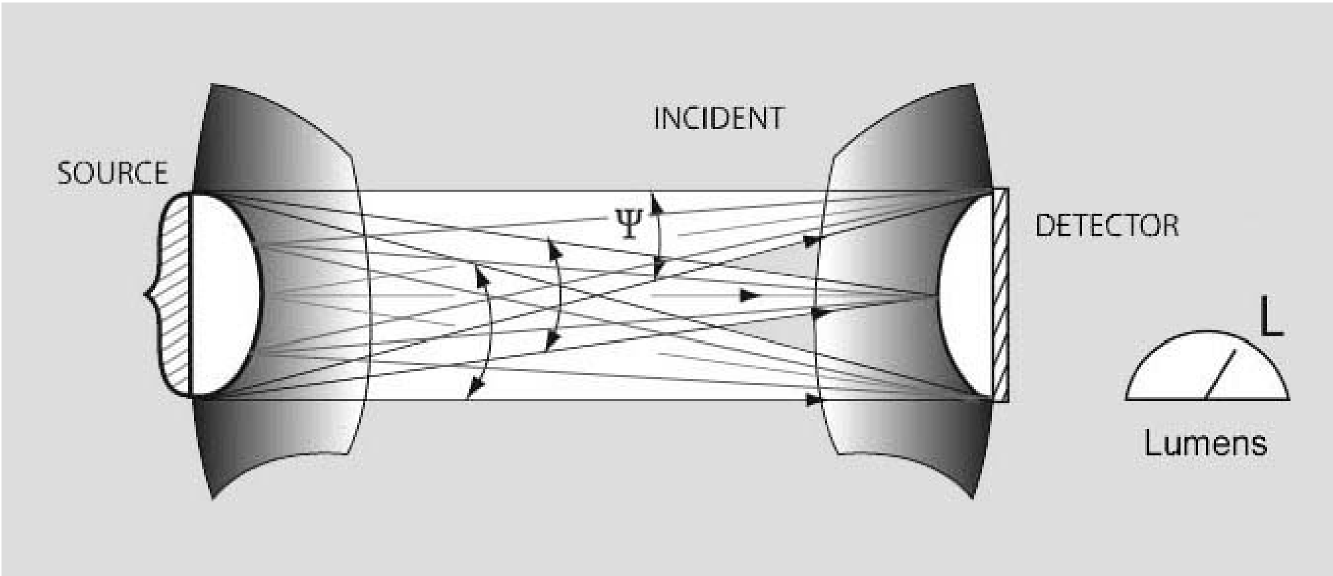


Figure 7. To measure incident luminance a detector's reading of L lumen is divided by the solid angle of the source and by the area of the detector.

The equation for incident luminance can be simplified by recognizing that the first part, "lumen / steradian", is the **candela** from measuring incident luminous intensity. Making that substitution results in:

incident luminance = L "candela" / meter^2 *detector area*

This looks just like the familiar equation for emitted luminance:

$$\text{luminance}_{emitted} = \text{L candela / meter}^2{}_{\text{source area}}$$

But they differ: 1) the incident measure divides the candela by area of the detector, while luminance divides by the area of the source; 2) the incident measure uses a solid angle based on the source, while luminance uses a solid angle based on the detector.

I now have some bad news and some good news:

SPECIFYING IMAGE LUMINOSITY

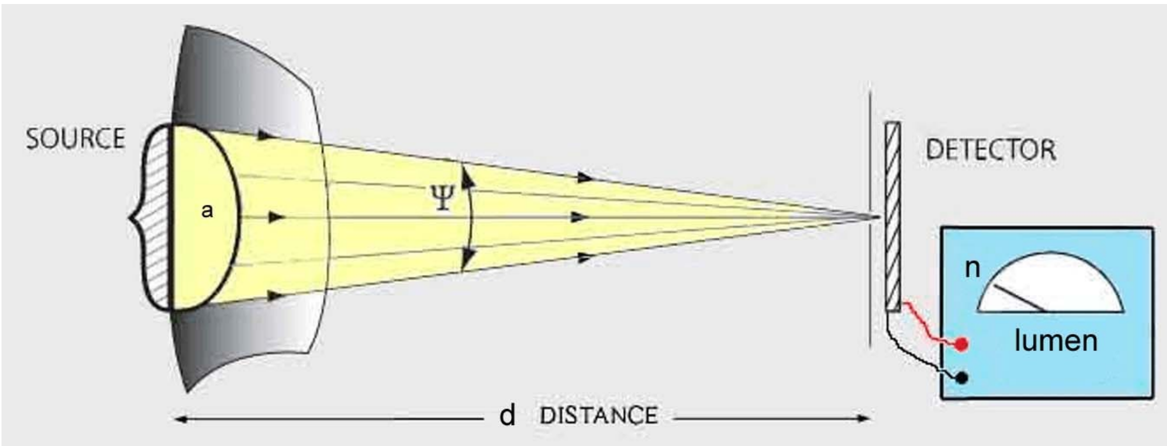
The bad news is that even incident luminance tells us little about the luminous effectiveness of an image. Retinas and other image sensing surfaces are not uniform detector surfaces. Their photoreceptor cells and sensors respond to images on an individual basis. While proportional to the average directional flux density that is luminance, a more direct approach is needed.

I propose **measuring the flux density actually impinging on the individual receptors**. The response from each of these is what determines the strength (brightness) of that segment (pixel) of the image. Here is how to do this without inserting a minature photometer into the focal plane of an eye or imaging device.

Pinhole Lens Images

To simplify the geometry involved, the explanation begins with an imaging device the uses a pinhole lens to produce the image.
Recall the model used to explain incident luminous intensity in Figure 6:

Fig. 6



After obtaining the reading of “n” lumen, moving the detector back results in an image of the source on the detector surface.
The amount of incident luminous intensity, “n” is not changed.
However, the detector in Figure 6 is changed in Figure 9 to an array receptors as would be the case in an imaging device like an eye or digital camera. The light source is Figure 9 is represented by a candle instead of a circular patch.

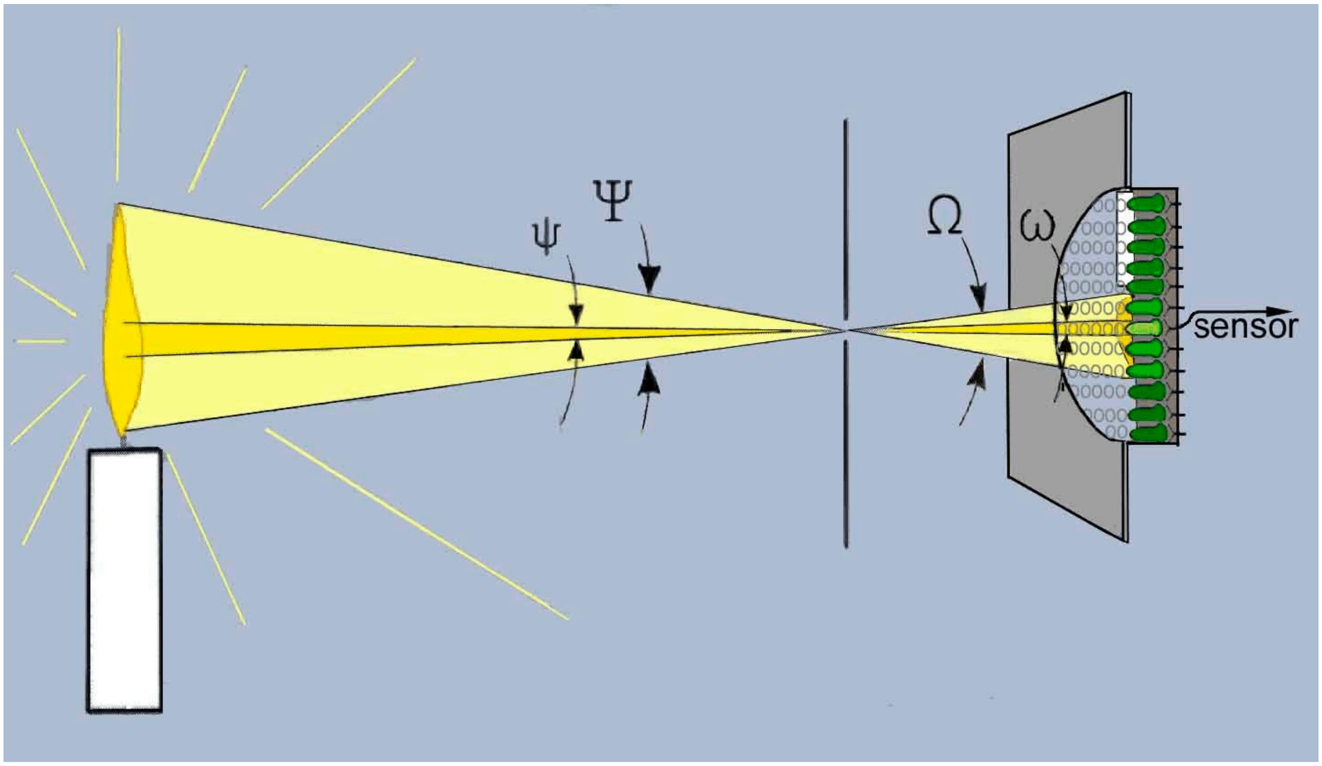


Figure 8. All of the image's light from the candle is contained within the incident solid angle Ψ steradians. This light is then emitted towards the receptors over Ω steradians. An individual receptor subtends ω steradians with respect to the pinhole and has an external receptive field of ψ steradians.

All the light received by a single sensor is contained within the solid angle ω . Since corresponding angles are equal, $\psi = \omega$ and $\Omega = \Psi$. Therefore, the receptor receives ω / Ψ th of all the incident light that produces the image.

Consider the following: 1) an imaging device using a pinhole lens, 2) an array of receptors each having a diameters of 0.1 mm, 3) the receptor array is located 10 cm behind the pinhole lens, 4) assume the flame represents a light source 10 cm in diameter at distance of 10 meters, 5) a detector placed directly behind the pin hole reads “n” lumen.

Each receptor will subtend a solid angle:

$$\omega = 10\ \text{mm}^2 / 10\ \text{cm}^2 = 10^{-8}\ \text{steradian}$$

The source subtends:

$$\Psi = 10\ \text{cm}^2 / 10^2\ \text{meters} = 10^{-4}\ \text{steradian}$$

The luminous flux per receptor = n * ω / Ψ = n*10^{-4} lumen

Note that the steradians cancel in the division. This leaves the measured lumen as the only remaining unit.

Focussed Images

Extending the pinhole imaging model helps seeing how to measure the images produced by an extended lens bringing many rays into focus. Figure 10 shows an extended lens focussing a candle flame onto an array of sensors. A detector with an aperture equal to the lens aperture placed at the lens' location reads L lumen. Given that the only available light is the candle, and neglecting intra-ocular losses, **L** represents all the luminous flux present in the image on the array. (The size of the aperture makes “L” much larger than the “n” reading obtained in the pinhole lens model.) Nevertheless, the portion of that flux which falls on a particular sensor is again a sensor's receptive field ω steradians divided by the solid angle Ψ steradians that the flame subtends with respect to the location of the lens.

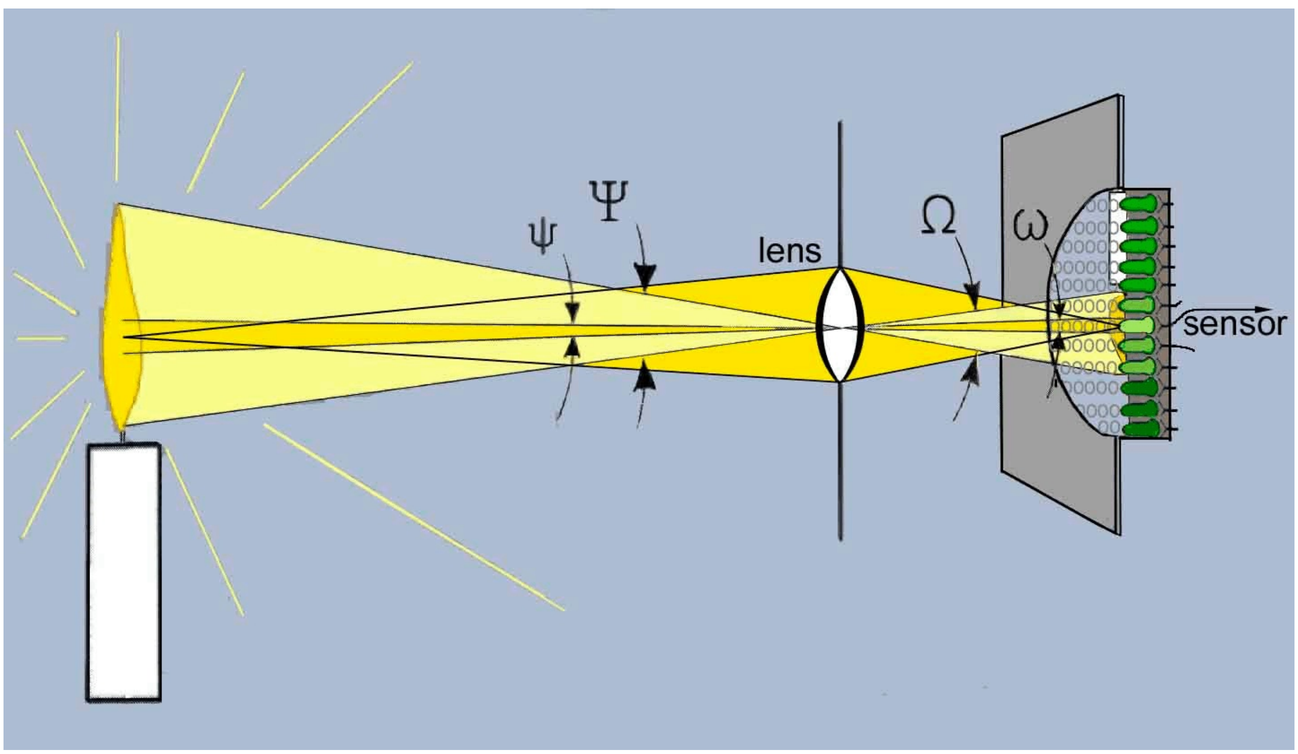


Figure 9. A candle's image is Focussed by an extended aperture lens on an array of sensors. All of the image's light from the candle is contained within the incident solid angle Ψ steradians. This light is then emitted towards the sensors over Ω steradians. An individual sensor subtends ω steradians with respect to the pinhole and has an external receptive field of ψ steradians.

As was the case with the pinhole lens model above, any particular sensor in that array receives one ω / Ψ th of all the light incident on the lens. If the individual receptors represented the receptor cells in the human fovea, they would have a receptive field ω of $1 * 10^{-8}$ steradians. The luminous flux on each receptor can then be calculated as:

$$\mathbf{L * (10^{-8} / \Psi) \text{ lumen}}$$

where Ψ is the solid angle of the image source with respect to the eye.

Again the steradians cancel in the ratio of the two solid angles leaving lumen as the only unit of measurement. Individual receptors do not care whether the image has been produced by a pinhole or extended lens.

What to call this measurement? Simply tell like it is:

Lumen per Receptive Field (l/rf)

Photonics and vision workers are more likely to have an illuminometer that reads lumen-per-meter² or a photometer that reads candelas-per-meter², than a calibrated lumen or watt meter with a variable aperture. Either of these can simplify the measurement procedure but require additional calculation. Using an illuminometer reading of **I** lumens-per-meter², with no other sources of light, the formula for lumens-per-receptive field becomes:

$$l/rf = \mathbf{I}_{lumen/meter^2} * \text{lens aperture}_{meter^2} * \omega / \Psi$$

where ω is the receptive field of a receptor.

If the image source is not uniform, one may want to calculate lumens-per-receptive-field only for those receptors that receive a certain portion of the image. With an illuminometer, lumen meter, or watt meter, this can be done by masking off the other parts of the image source. Using round or square masks simplifies calculating the solid angle of that portion.

Some focusing photometers have measuring fields small enough to select certain portions of an image source without further ado. Also with such photometers, eliminating all other sources of light is less critical and the solid angle of the image source is defined. The "candelas" in the candelas-per-meter^2 luminance unit is a measure of source flux per solid angle. For a photometer reading of **L** candelas-per-meter^2, the calculation of lumens-per-receptive-field is:

$$l/rf \ = \ \boldsymbol{L}_{(lumen/steradian)/meter^2} * \text{lens aperture}_{meter^2} * \omega_{steradians}$$

where ω is the receptive field of a receptor.

CONVERTING TO INCIDENT PHOTOMETRY

I hope that everyone who measures optical radiation for vision research or other imaging devices now wonders: "Where can I get an incident light photometer?" "Is it possible to convert my present photometer to measure incident luminance?"

Here is the **good news**. It **is** possible to convert your photometers. Here is how to do it:

We start with the non-simplified definition of regular (emitted) luminance as a detector's lumen reading / the detector solid angle with respect to the source / source area:

$$\text{luminance}_{emitted} = \text{lumen} / (\textbf{steradians}_{\textit{detector}}) / \text{area}_{source}$$

Breaking down the enigmatic solid angle that the detector subtends with respect to source in terms of the detector area and its distance from the source gives:

$$\text{luminance}_{emitted} = \text{lumen} / (\textbf{area}_{\textit{detector}} / \textbf{distance}^{\mathbf{2}}) / \text{area}_{source}$$

Rearrange the divisions, and we have:

$$\text{luminance}_{emitted} = \text{lumen} * \text{distance}^2 / (\text{area}_{detector} * \text{area}_{source})$$

Following the same steps with incident luminance produces:

$$\text{luminance}_{incident} = \text{lumen} * \text{distance}^2 / (\text{area}_{detector} * \text{area}_{source})$$

Note that the components of the equations for incident luminance are the same as those for emitted luminance.

The two measurements are equal!

So you can convert your old emitted-light photometers to measuring incident luminance. Just follow these steps:

1. Print the pdf file of this presentation.
2. Clip out the following tag.
3. Paste it on your photometer.

THIS PHOTOMETER MEASURES INCIDENT LUMINANCE

Now that you know incident and emitted luminance measures are the same. You may find the following disclosure helpful.

Photoelectric sensors only respond to the rate of photon impingement (or lumen) over their surface area. Illuminometers are calibrated in terms of their sensor area to measure illuminance in **lumen/meter^2**. Photometers add the steradian division step in terms of the solid angle defined by their optics to obtain the **candela/meter^2**.

N.B. *It is the optics that make photometers expensive. Here is how to skip the optics and still measure luminance:*

1. *Measure illuminance with a simple illuminometer.*
2. *Then measure the solid angle of the source with respect to the illuminometer.*
3. *Finally, do the division by hand:*

luminance = illuminance / solid angle of the source

Another Measurement to Consider

As presented, the above measurements do not take into account internal factors which can diminish the amount of light that reaches the sensor array after the light has entered the imaging device. Such losses may be estimated by various means and used to correct the above calculations. That still leaves the question of sensor sensitivity. This can be approached by comparing the output of different devices and arrays in the presence of the same image source. In human vision, "outputs" ranging from cortical evoked potentials to reaction times have been used. Yet such measurements will estimate only relative sensitivity. A completely different photometric approach used by vision researchers for hundreds of years, but rarely today, enables estimating the absolute sensitivity of receptors.

Using an image source of standard luminance, one attenuates the available light with neutral density filters until the sensor response ceases or reaches the noise level of the system. That is called the system's "threshold". Sensor sensitivity is then defined in terms of the attenuation - usually in log units below the standard luminance. To study and specify the operation of sensor arrays, the use of both lumens-per-receptive-field and threshold approaches holds promise for obtaining a more accurate description of how imaging systems work.

For further explanation, a bit of history, and references see: T. Nilsson (2009) Photometric specification of images. *Journal of Modern Optics*, 56, 1523-1535.