Analysis of Sensitivities and Diffraction Corrections in Radiometry

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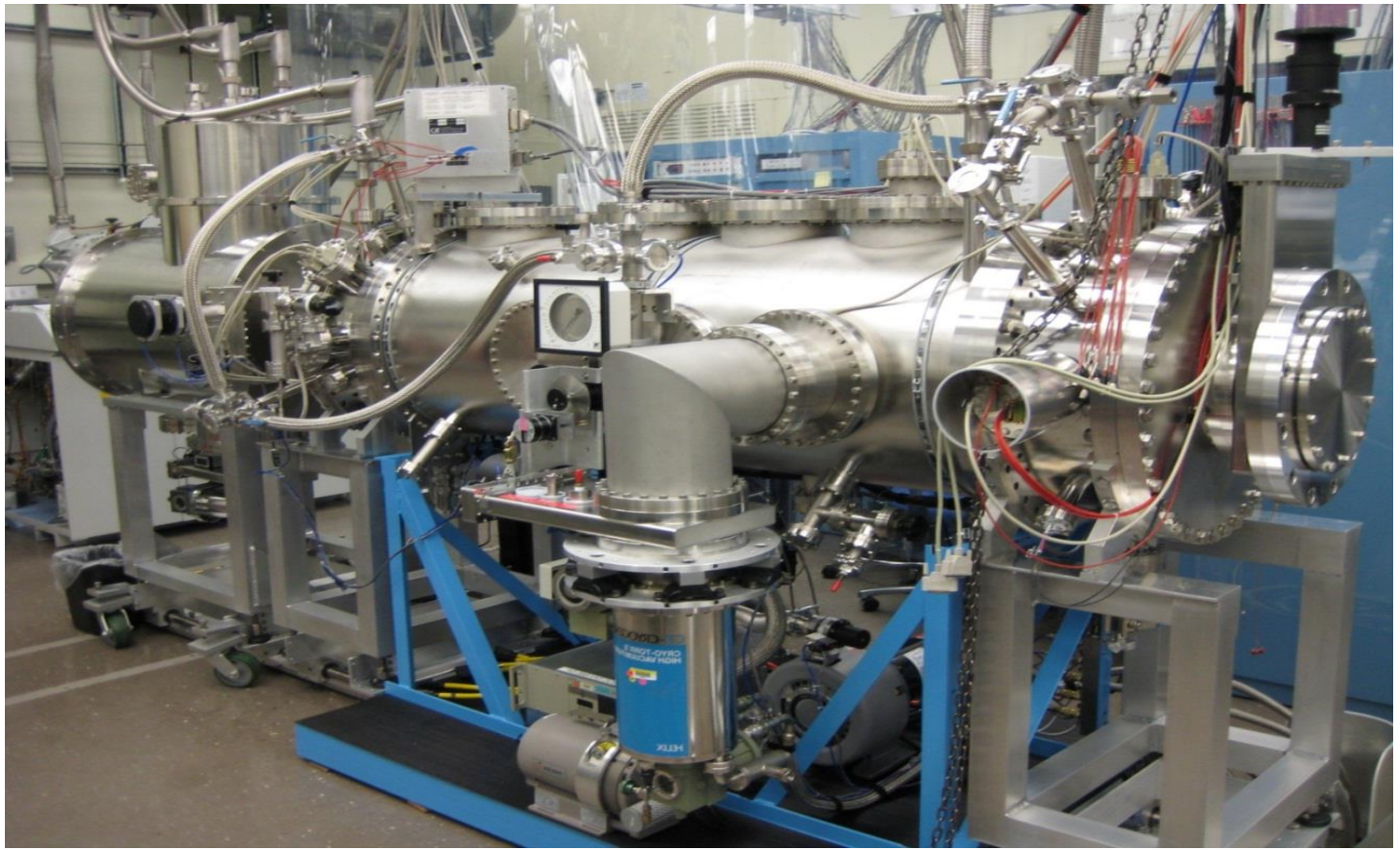
- Diffraction effects can be substantial in radiometry
- Numerical calculations with small uncertainty are available
- Model calculations are available that work well

Broadband Calibration Chamber (BCC)

NIST Low-Background Infrared (LBIR) Facility



"Blackbody calibration" measures *radiance temperature*

It involves the entire Planckian spectrum

$$L(\lambda,T)=\frac{c_{1L}}{n^2\lambda^5\left(\exp\left(\frac{h}{n\lambda\, c\, kT}\right)-1\right)}$$

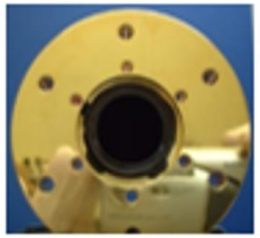
$$L(T)=\int_0^\infty d\lambda\, L(\lambda,T)$$

$$L(T)=\frac{2\pi^5}{15}\frac{k^4T^4}{c^2h^3}\frac{1}{\pi}=\sigma T^4\frac{\cos(\theta)}{\pi}$$

The Broadband Calibration Chamber calibrates customer blackbodies using an *Absolute Cryogenic Radiometer (ACR, a.k.a. Active-Cavity Radiometer)*

Blackbody calibration, chamber model:

NIST Primary Optical Watt Radiometer (POWR)



Active-Cavity Radiometer (ACR)
(held at constant, liquid-He temperature, shuttered, compares Resistive (ohmic) heating power to optical power received)

Blackbody defining aperture (BBDA)

resistor

Blackbody cavity

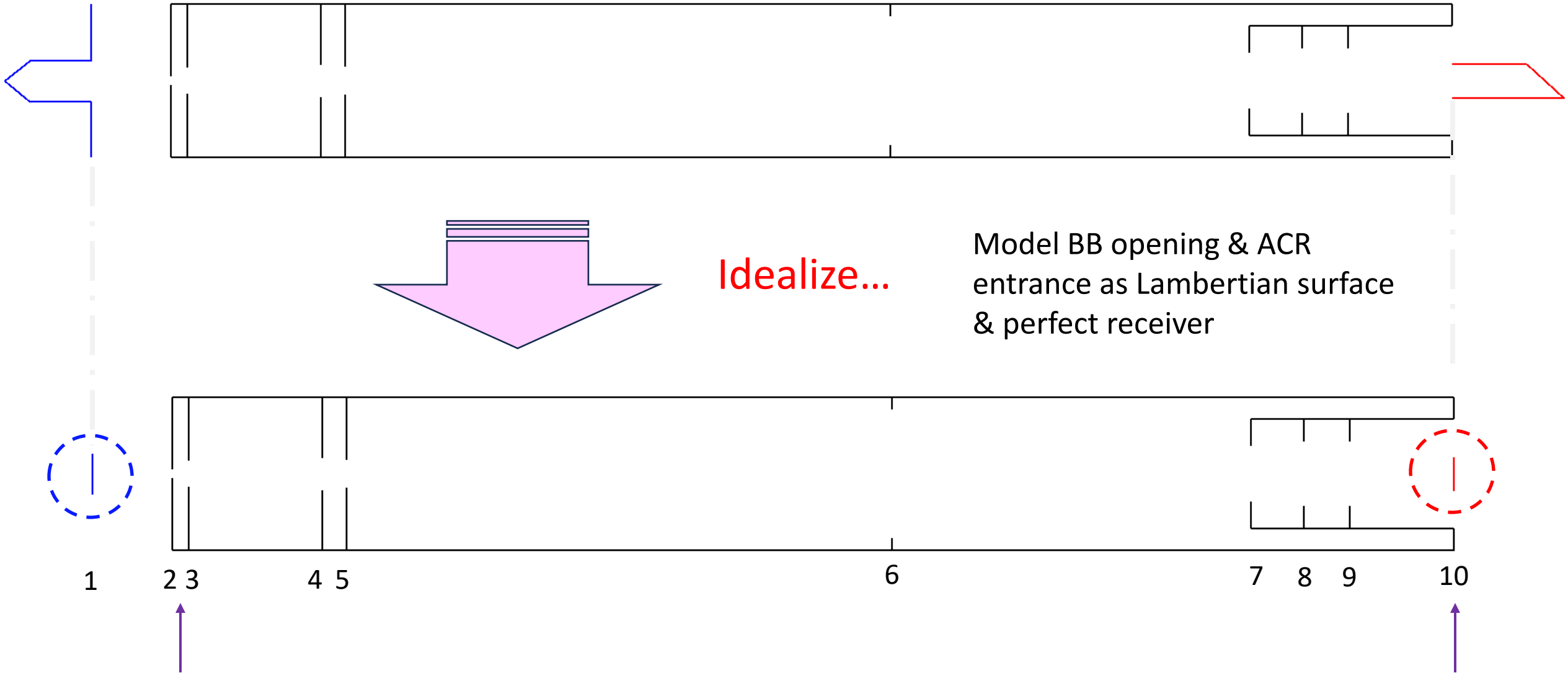
HEAT SINK

8 intervening optical elements, restricting & diffracting light

$T < 800$ K

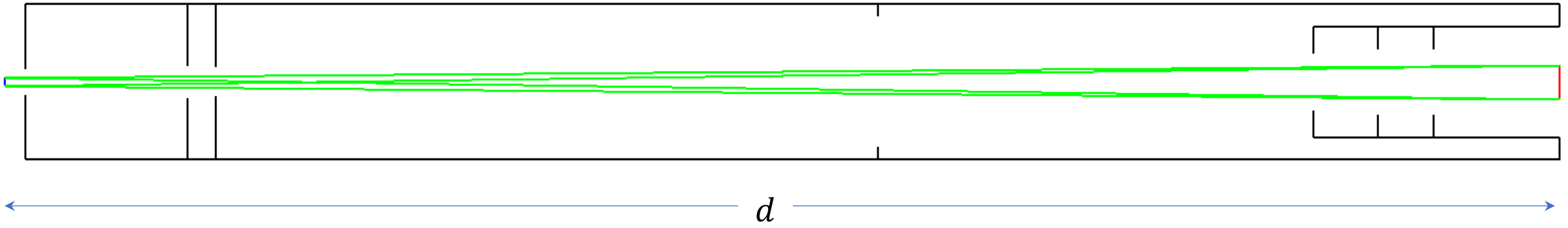
d

Blackbody calibration, chamber model:



Note: Apertures 2 & 10 are precision apertures.

Combination 2-10 defines geometrical throughput and expected power.

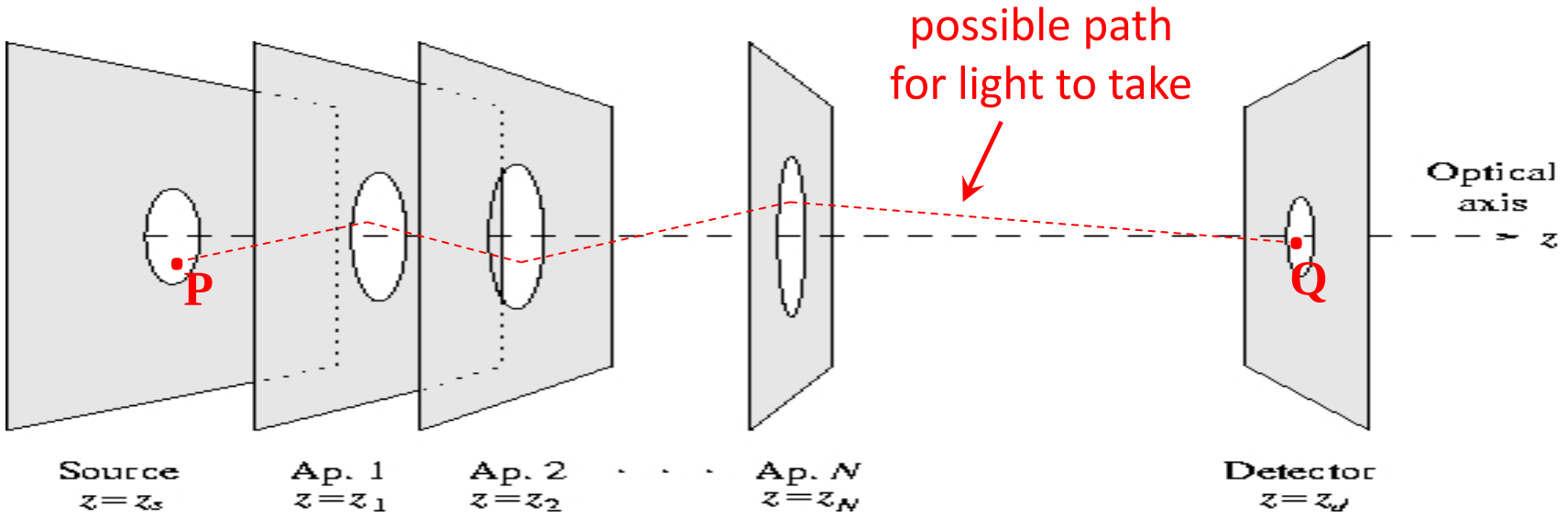


$$\Phi = L \cdot \frac{\pi^2}{2}\left\{d^2 + R_{\mathrm{BBDA}}^2 + R_{\mathrm{ACR}}^2 - \left[\left(d^2 + R_{\mathrm{BBDA}}^2 + R_{\mathrm{ACR}}^2\right)^2 - 4R_{\mathrm{BBDA}}^2 R_{\mathrm{ACR}}^2\right]^{1/2}\right\}$$

$$= L \cdot \frac{\pi^2}{2}\left(\frac{4R_{\mathrm{BBDA}}^2 R_{\mathrm{ACR}}^2}{d^2 + R_{\mathrm{BBDA}}^2 + R_{\mathrm{ACR}}^2 + \left[\left(d^2 + R_{\mathrm{BBDA}}^2 + R_{\mathrm{ACR}}^2\right)^2 - 4R_{\mathrm{BBDA}}^2 R_{\mathrm{ACR}}^2\right]^{1/2}}\right)$$

$$\cong L \cdot \left(\frac{\pi R_{\mathrm{BBDA}}^2 \cdot \pi R_{\mathrm{ACR}}^2}{d^2}\right)$$

Consider an optical system, and the *wave propagation* of a light field *u* from P to Q:



Maxwell's equations→scalar Kirchhoff diffraction theory.
Repeated application of Kirchhoff's diffraction formula gives

$$u(\mathbf{Q}) = \frac{U_0}{(i\lambda)^N} \int_{A_1} dx_1 dy_1 \ldots \int_{A_N} dx_N dy_N \; G(\mathbf{P}, \mathbf{x}_1) \cdots G(\mathbf{x}_N, \mathbf{Q}) \cdot \exp[\, i \sum_k \delta l_k(\mathbf{x}_k)]$$

For powered optics (not in this work)

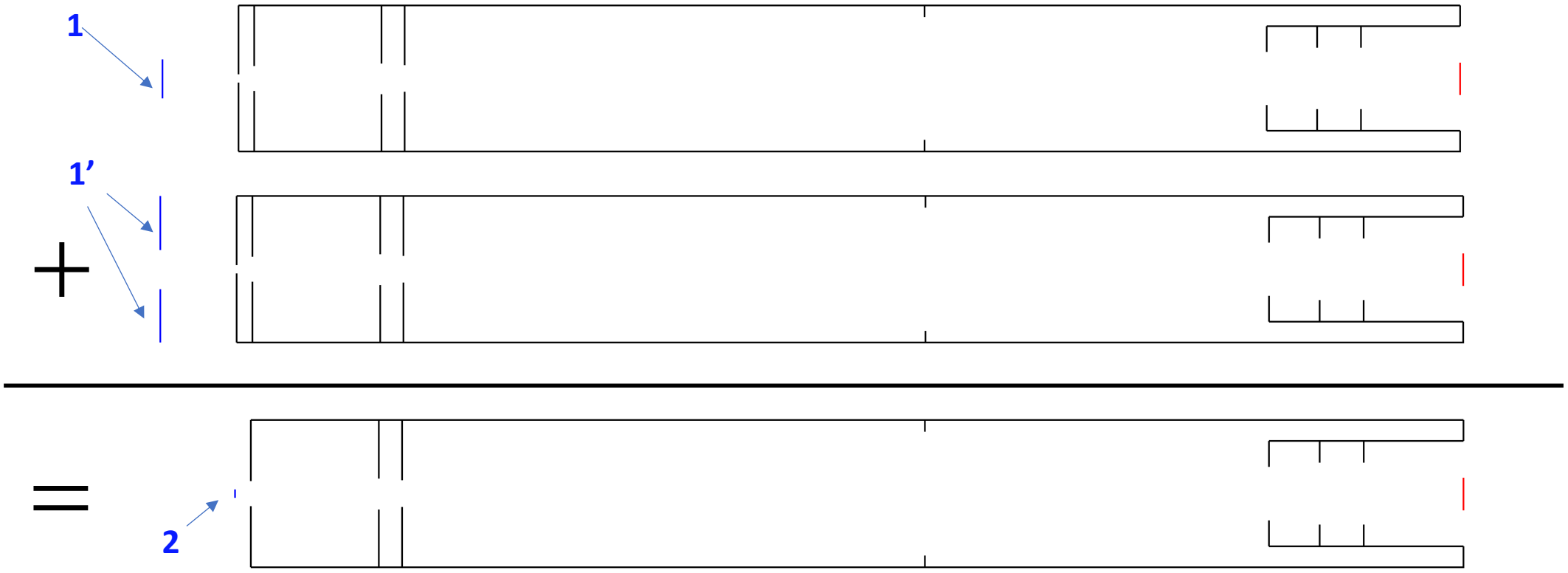
Numerical computational cost $\propto 1/\lambda^3$ *
Dense sampling required at small λ, so…

→ Approximate models at small λ
Numerical methods used at large λ

* See Rubin et al., Appl. Opt. **57**, 788 (2018)

Flux conservation:* having Aperture 2 be an ideal Lambertian emitter equivalent to having *entire plane* of BB opening with equivalent radiance.

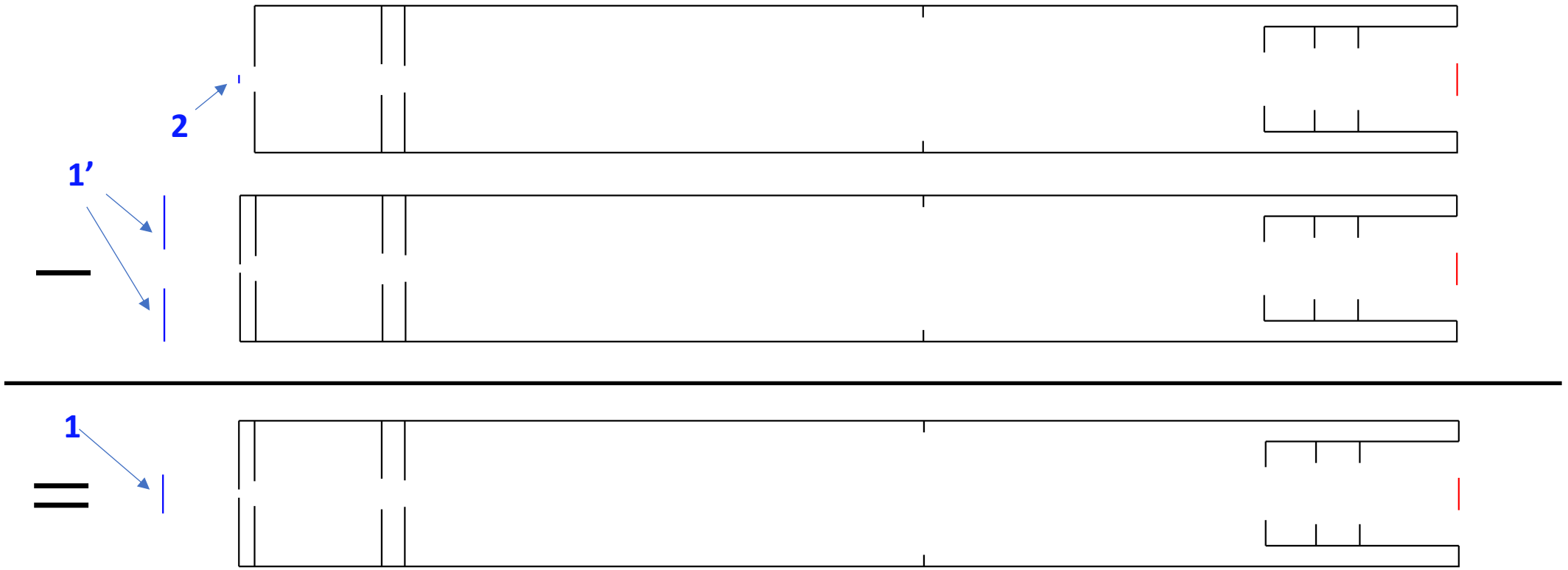
Call the BB opening, **1**, and its complement, **1'**



Functionally, one has **1 + 1' = 2**

* J. W. Goodman, *Introduction to Fourier Optics*

Flux conservation:* having Aperture 2 be an ideal Lambertian emitter equivalent to having *entire plane* of BB opening with equivalent radiance.



Functionally, we have **1 = 2 – 1’**

$$a + b = c \quad \Longrightarrow \quad b = c - a$$

* J. W. Goodman, *Introduction to Fourier Optics*

Modeling SAD Subsystems

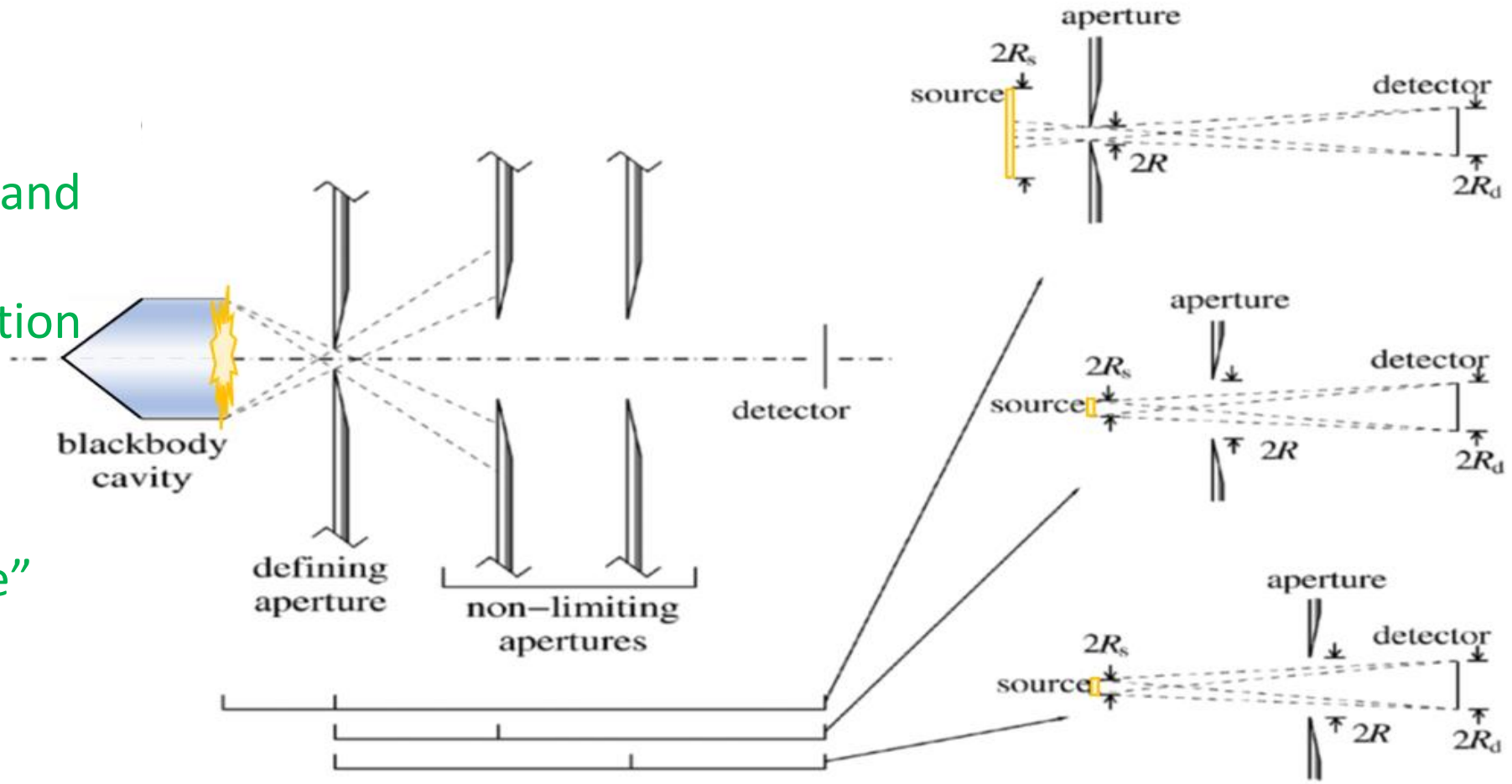
Largest effects can be described by identify three-element source-aperture-detector (SAD) subsystems *

* See JOSA A **33**, 1509 (2016), and references therein
(Wolf, Lommel, Blevin, Sanders and Jones, Ooba, Steel, De and Bell, Edwards & McCall)

Example:

Five element systems, and three SAD subsystems exhibiting main diffraction effects.

Note “extended source”

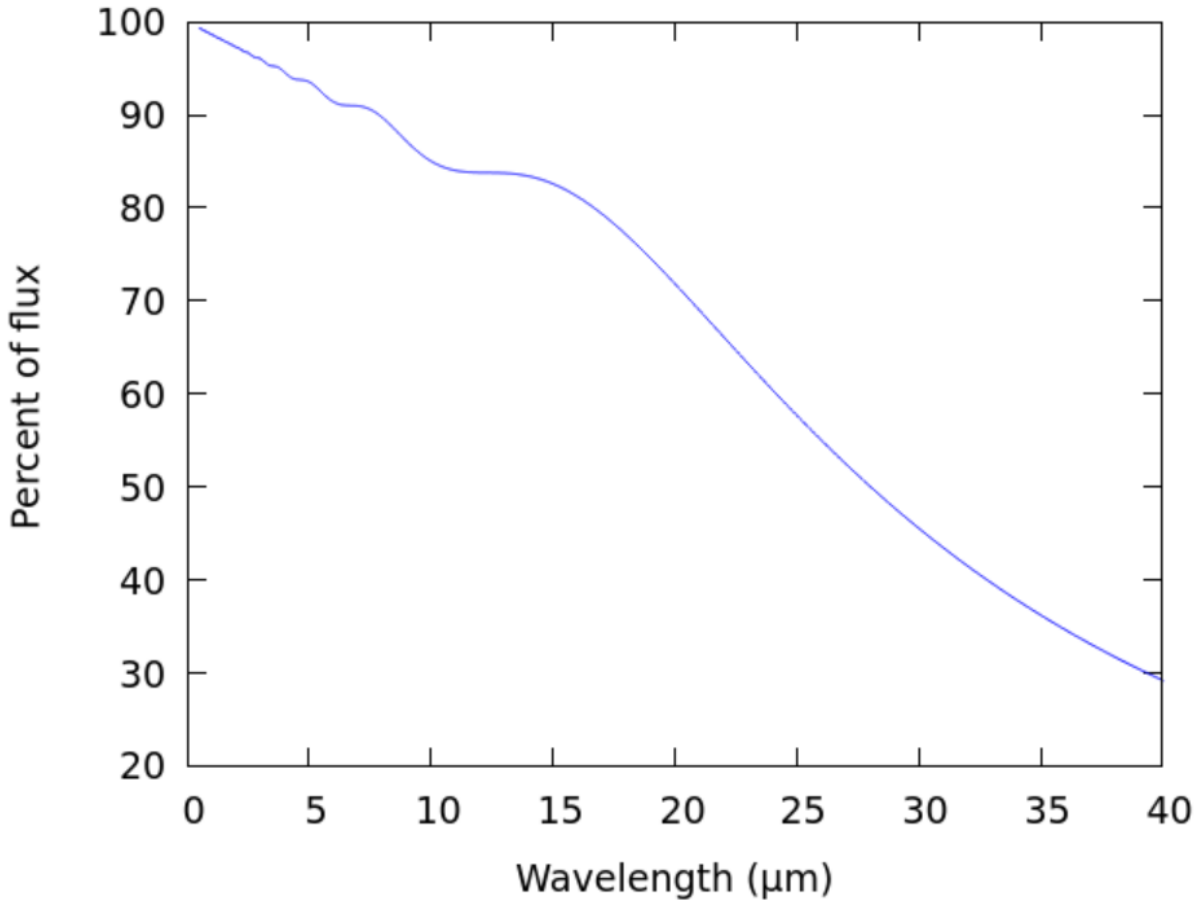


SAD Combination 1-2-10 leads to a diffraction loss (estimated by 1’-2-10 “false” light).

SAD 1 2 10

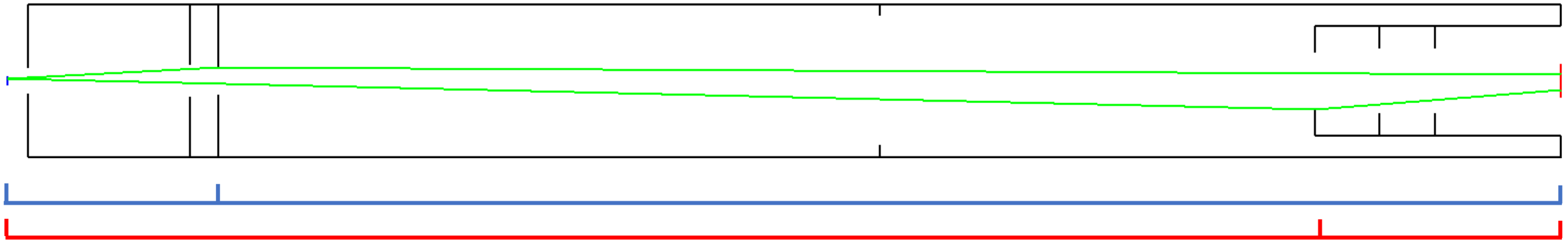
SAD subsystem scales spectral power at ACR by a factor shown:

SAD effects given by Wolf’s formula for integrated flux + accounting for “extended source.”

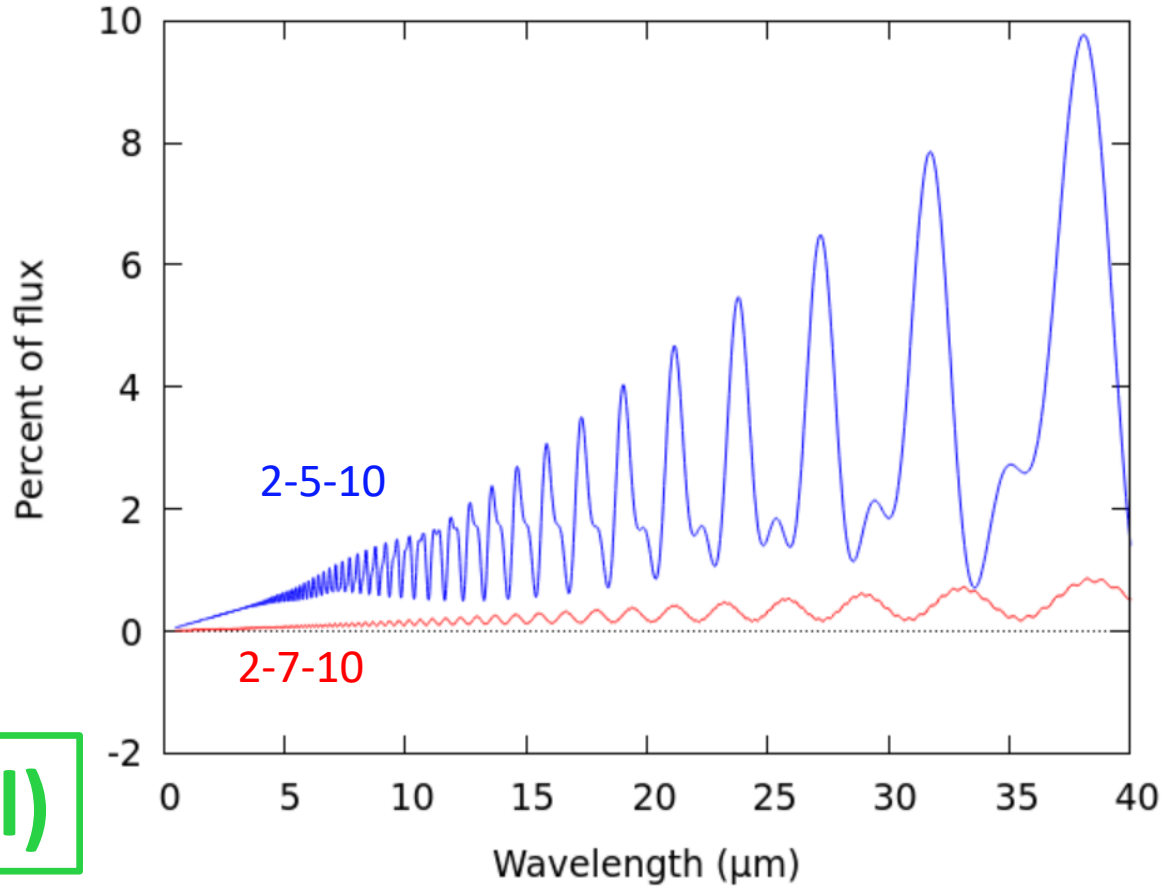


0-baffle effect

SAD Combinations 2-5-10 and 2-7-10 leads to first-order diffraction gains.



SAD subsystems increase spectral power at ACR by amounts shown:



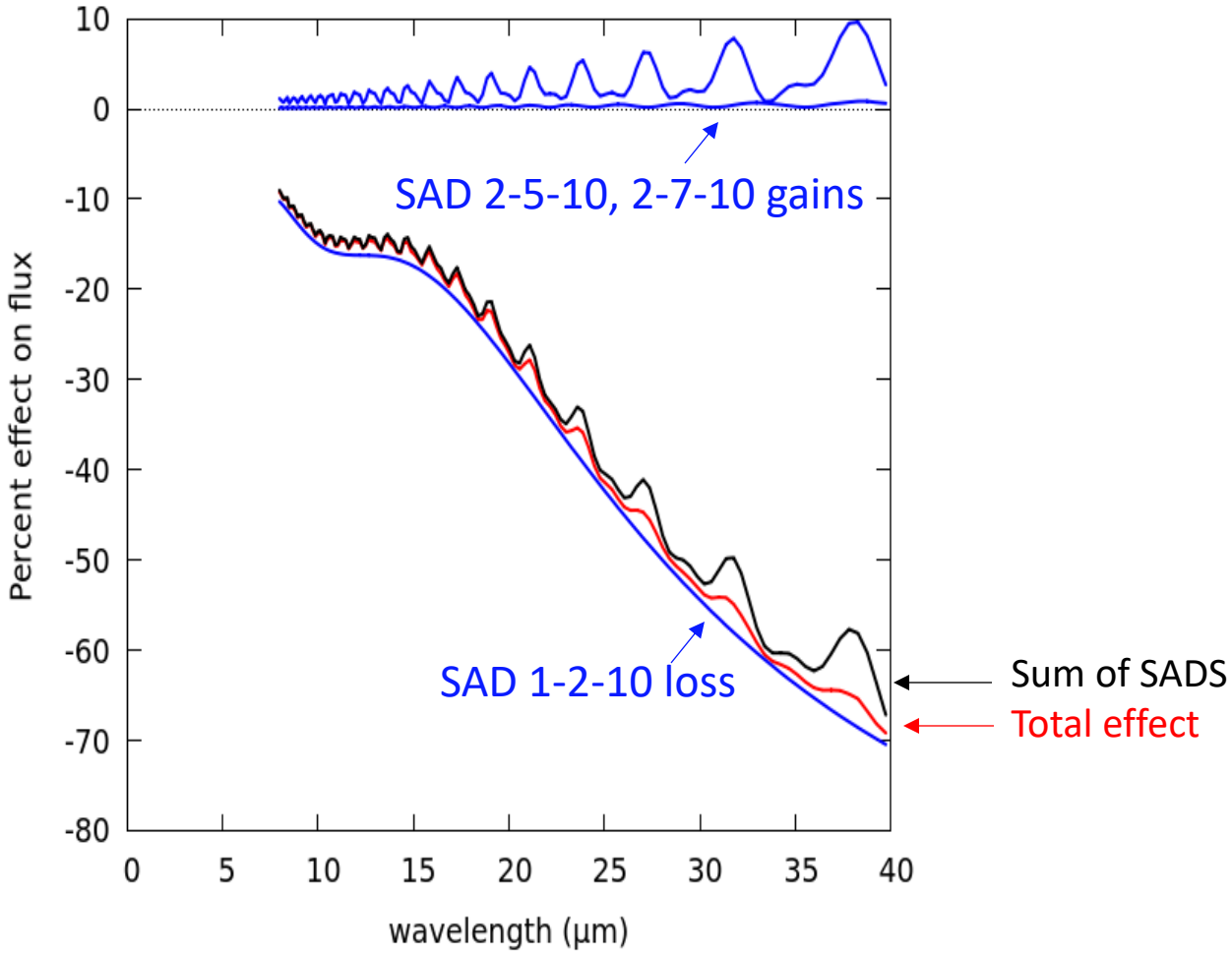

```
SAD  2  5  10
SAD  2  7  10
```

1-baffle effect (partial)

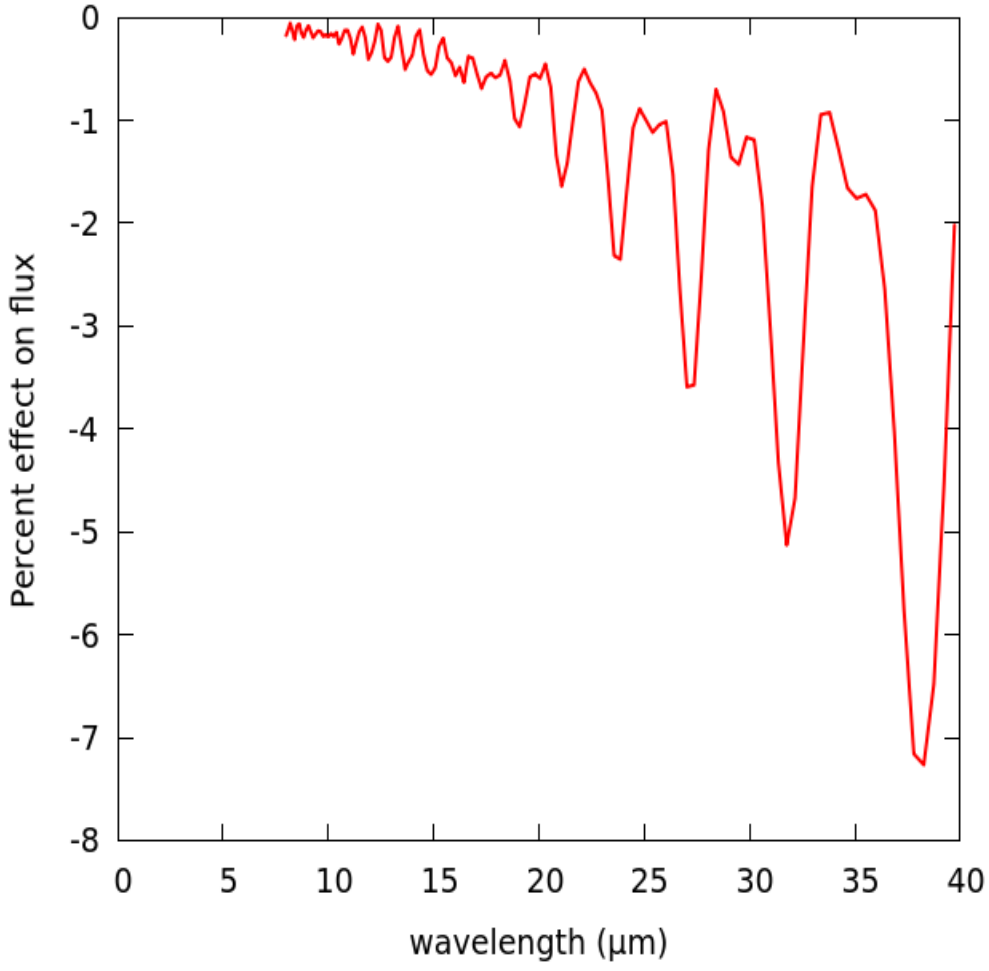
Combination of SAD results versus total effect

```
SAD  1  2  10
SAD  2  5  10
SAD  2  7  10
```

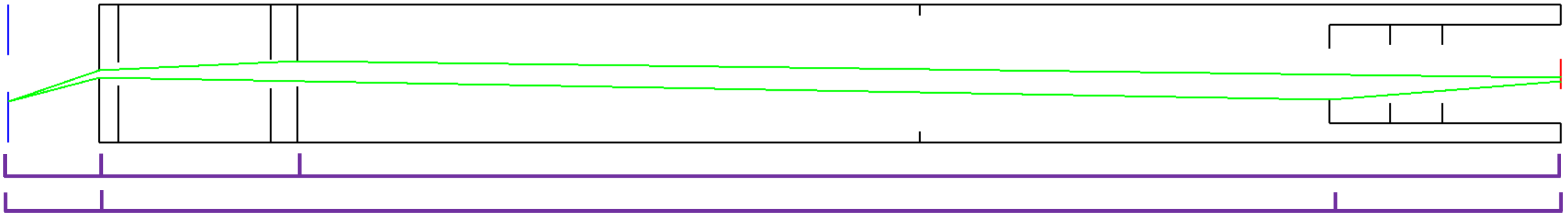
SAD Effects vs Total Effect



"Beyond-SAD" Effects



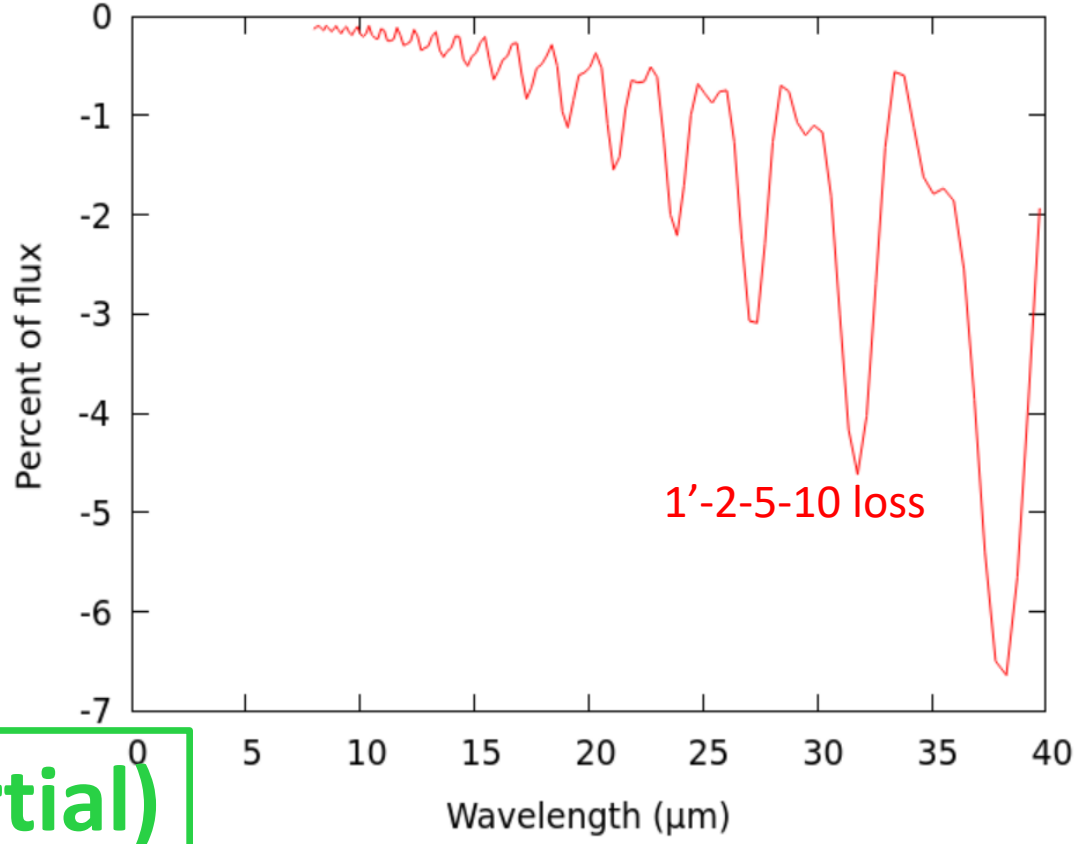
Combinations 1’-2-5-10 and 1’-2-7-10 account for effective decreases of 2-5-10 and 2-7-10 gains.



```
OFB  1  2  5  10
OFB  1  2  7  10
```

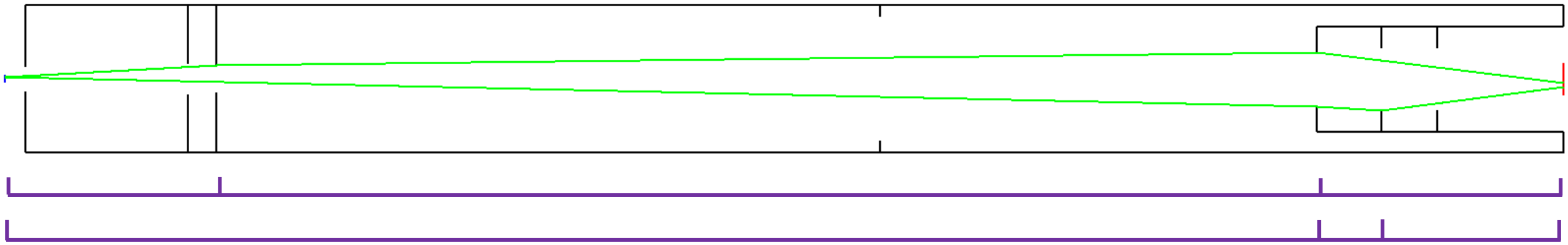
Example:
1’-2-5-10 loss is shown

Exact formulas for this *largest beyond-SAD* term are now in-hand.



1-baffle effect (partial)

Combinations 2-5-7-10, 2-7-8-10, … leads to 2nd-order diffraction gains.
Also correctible for loss at BBDA (not shown).



```
XBP  2  5  7  10
XBP  2  7  8  10
XBP  2  7  9  10
```

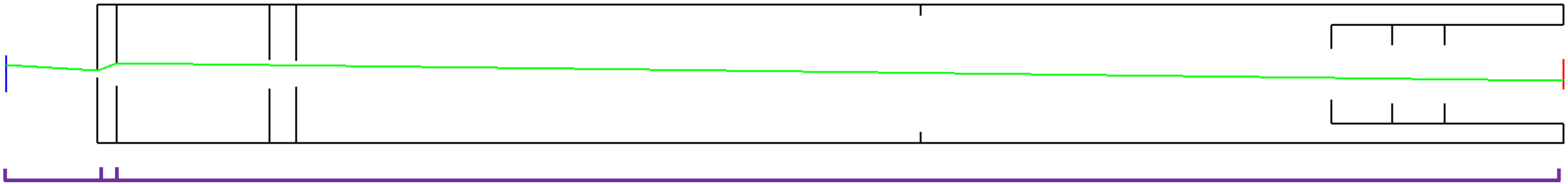
These gains vary roughly as λ^2 or $1/T^2$.* Also, vignetting can also affect such gains.

* Coefficient of λ^2 (and thus $1/T^2$) found for several 2nd-order effects in JOSA A **42**, 476 (2025)

2-baffle effect

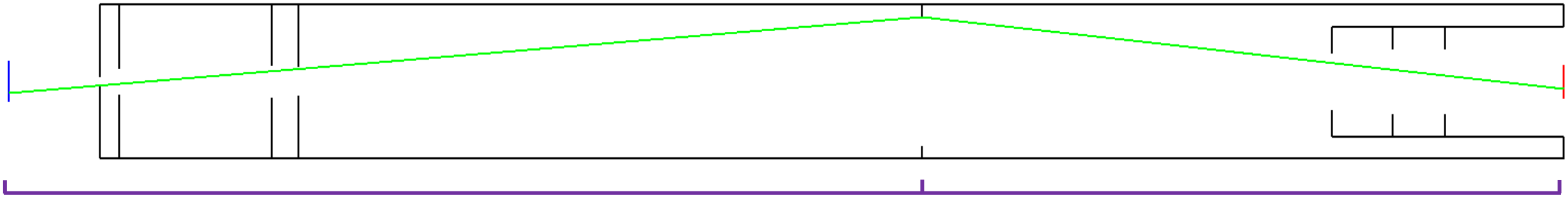
Sundry additional “beyond-SAD” corrections…

1-2-3-10 2nd-order diffraction leads to a small accidental gain.



```
UFB   1   2   3   10
```

For larger Aperture 2, even the very large Aperture 6 can lead to a small 1st-order gain.



2-baffle effect

```
SPC1    1    6   10          5.592497        7.017125        -4.371048        9.996000
```

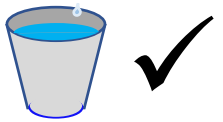
Sample 2-baffle contributions to diffraction correction (ppm), and model error

Contribution	$T = 200$ K	$T = 500$ K
σ_3'	-3399	-613
σ_5'	-285	-52
δ_{23}	31 (0.9)	4 (0.1)
δ_{35}	40 (0.9)	7 (0.1)
δ_{56}	19 (0.0)	4 (0.0)
δ_{57}	79 (0.0)	20 (0.0)
$\delta_4 + \delta_{45}$	1992 (0.1)	845 (0.0)
δ_{24}	0	0
δ_{34}	10	2
δ_{37}	5	1
δ_{46}	0	0
δ_{47}	0	0

Model=exact

Model result, model error

Model result only



Sample 0-, 1- & 2-baffle diffraction contributions, model-exact total discrepancy shown

T (K)	E_0	E_1	E_2	$\psi_{\mathrm{tot}} - 1$	Discrepancy
200	−45240	30911	171	−14159	−0.4
250	−36134	25330	117	−10687	−0.3
300	−30083	21446	85	−8551	0.5
350	−25770	18591	65	−7114	0.5
400	−22539	16404	52	−6083	0.4
450	−20029	14676	42	−5310	0.4
500	−18022	13277	35	−4709	0.4

3- and beyond-3-baffle effects appear inconsequential.

CONCLUSIONS

- Diffraction effects can be substantial in radiometry
- Numerical calculations with small uncertainty are available
  - Rubin et al., Appl. Opt. **57**, 788 (2018)
  - Shirley et al., 2002 Meas. Sci. Conf. Proceedings (2002)
- Model calculations are available that work well are available
  - Shirley, E.L., to be submitted.